Heat Transfer

Unit V
Heat Exchangers

Heat Exchangers

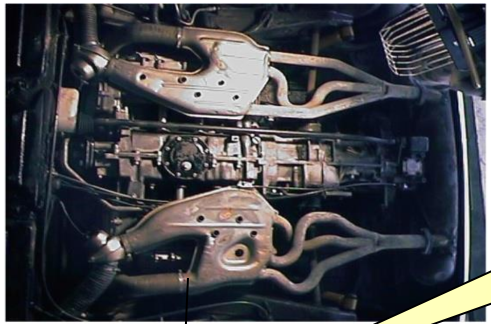
- Heat exchanger is an equipment, in which transfer of heat energy takes place from hot fluid to cold fluid.

Examples are:	
	Automobile Radiators
	Preheaters
	Intercoolers
	Boilers
	Condensers
	Oil coolers
	Cooling Towers

- Some manufactures:

  Thermax, Forbes Marshall, TATA, Behr, Alfa Laval, Paharpur

Applications of Heat Exchangers



Heat Exchangers prevent car engine overheating and increase efficiency

Heat exchangers are used in AC

Heat exchangers are used in chemical Industry for heat transfer

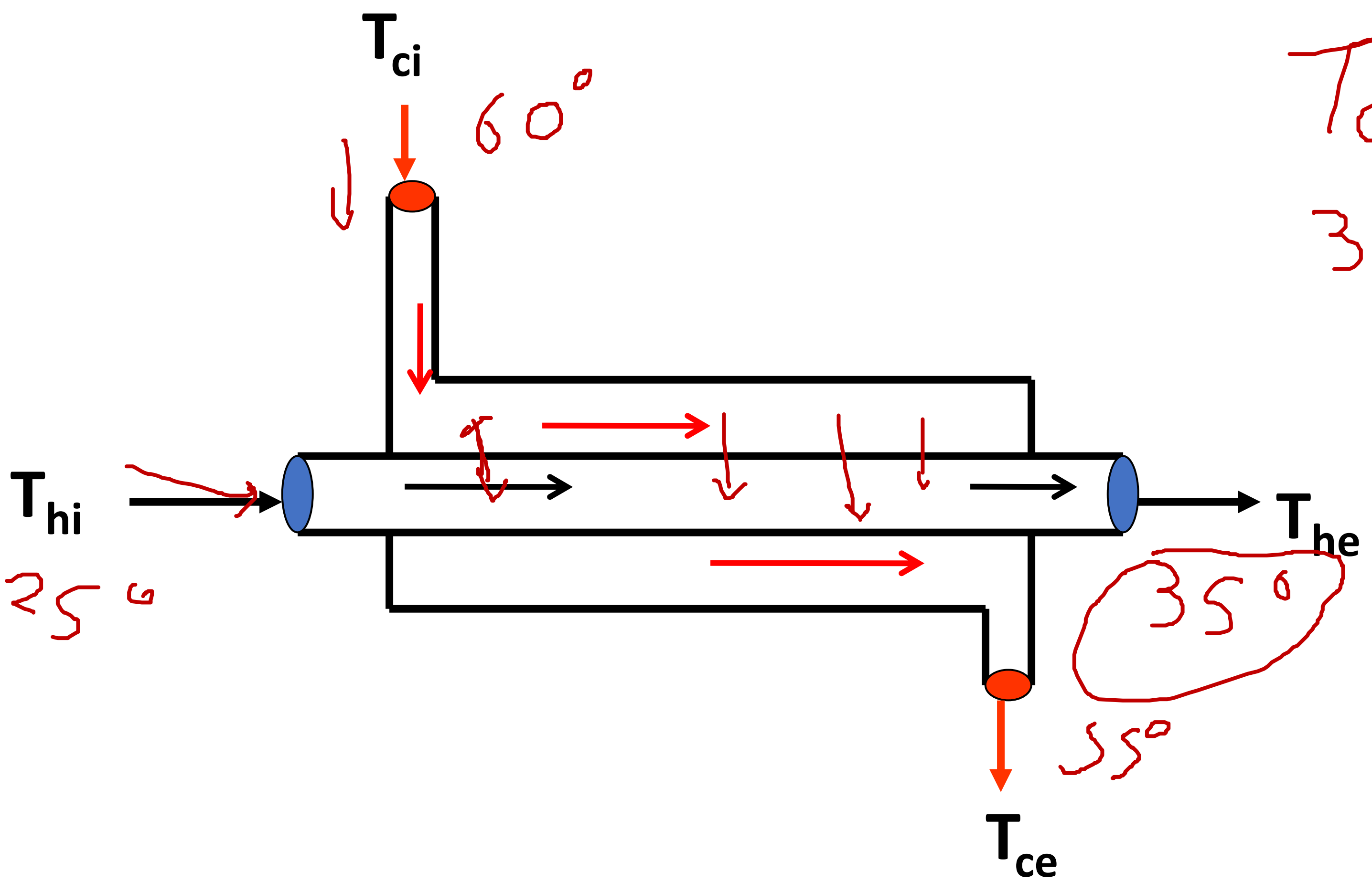
Types of Heat Exchangers

Direct Transfer type (Recuperator):
- Automobile Radiators, Oil Coolers,
- Air preheaters,
- Super heaters, Condensers, Evaporators etc.

Storage Type (Regenerator):
- Open hearth and glass melting furnaces,
- Air heaters of Blast furnaces

Direct Contact Types:
- Cooling Towers, Jet condensers

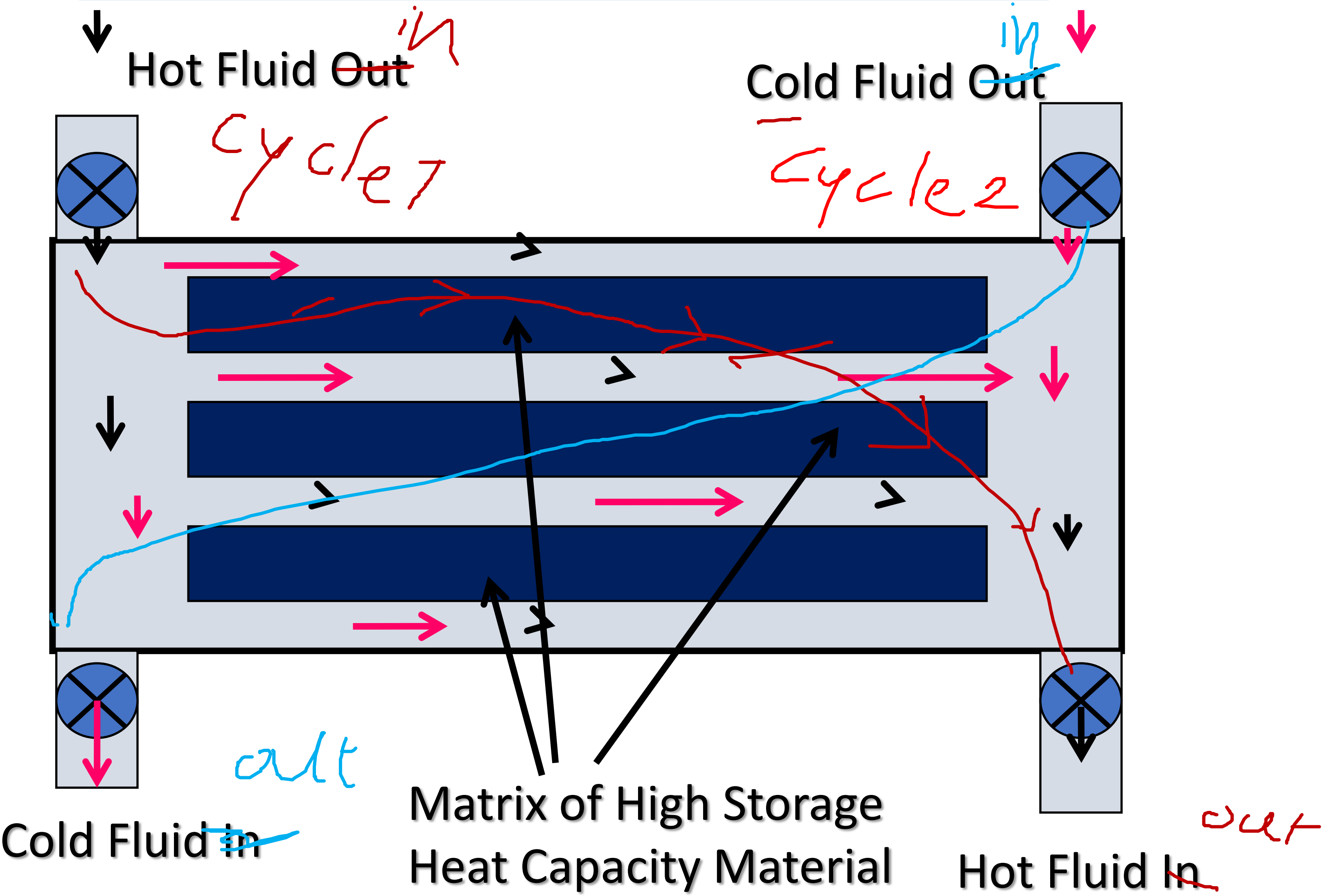
Direct Transfer Type Heat Exchanger



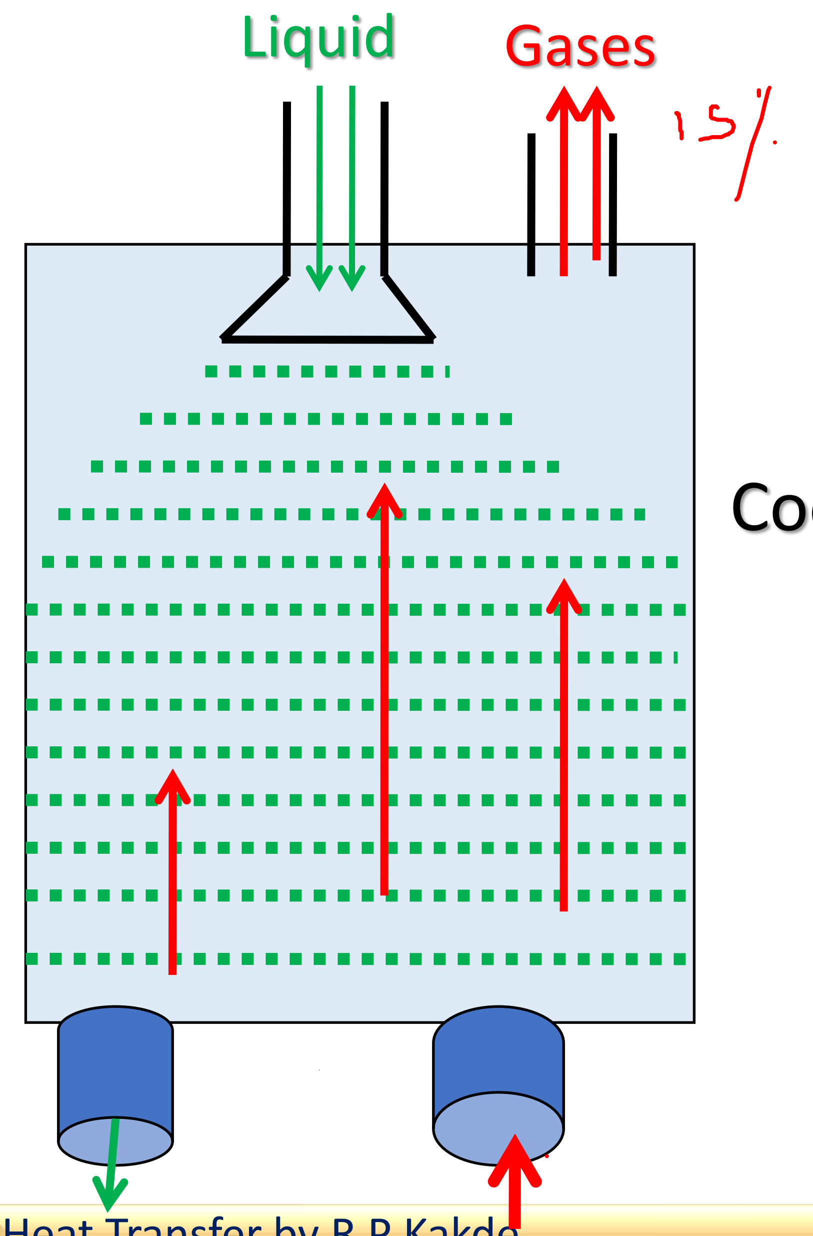
$T_{ce} \leq T_{he}$

35 < 55

Storage Type Heat Exchanger



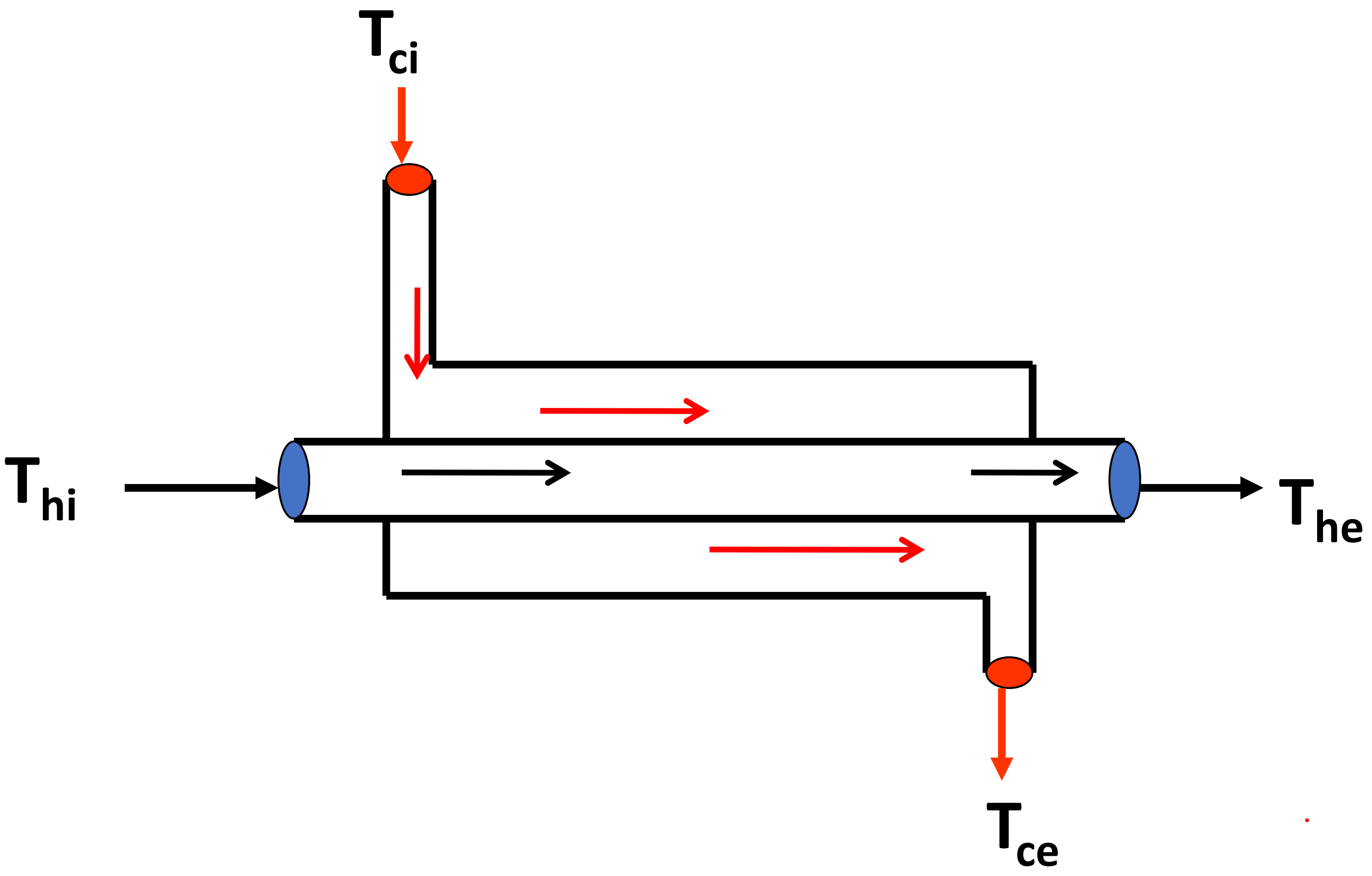
Direct Contact Type Heat Exchanger



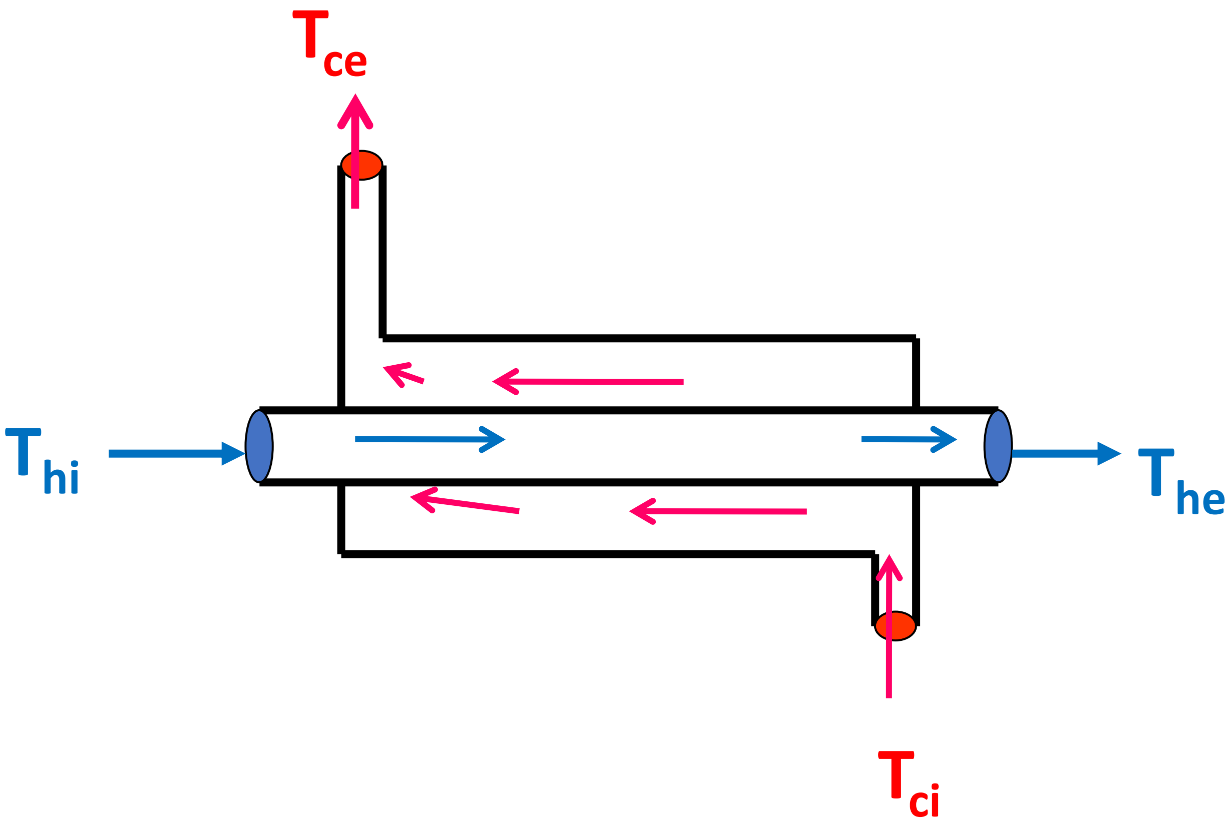
Cooling Tower

Direct Transfer Type Heat Exchanger

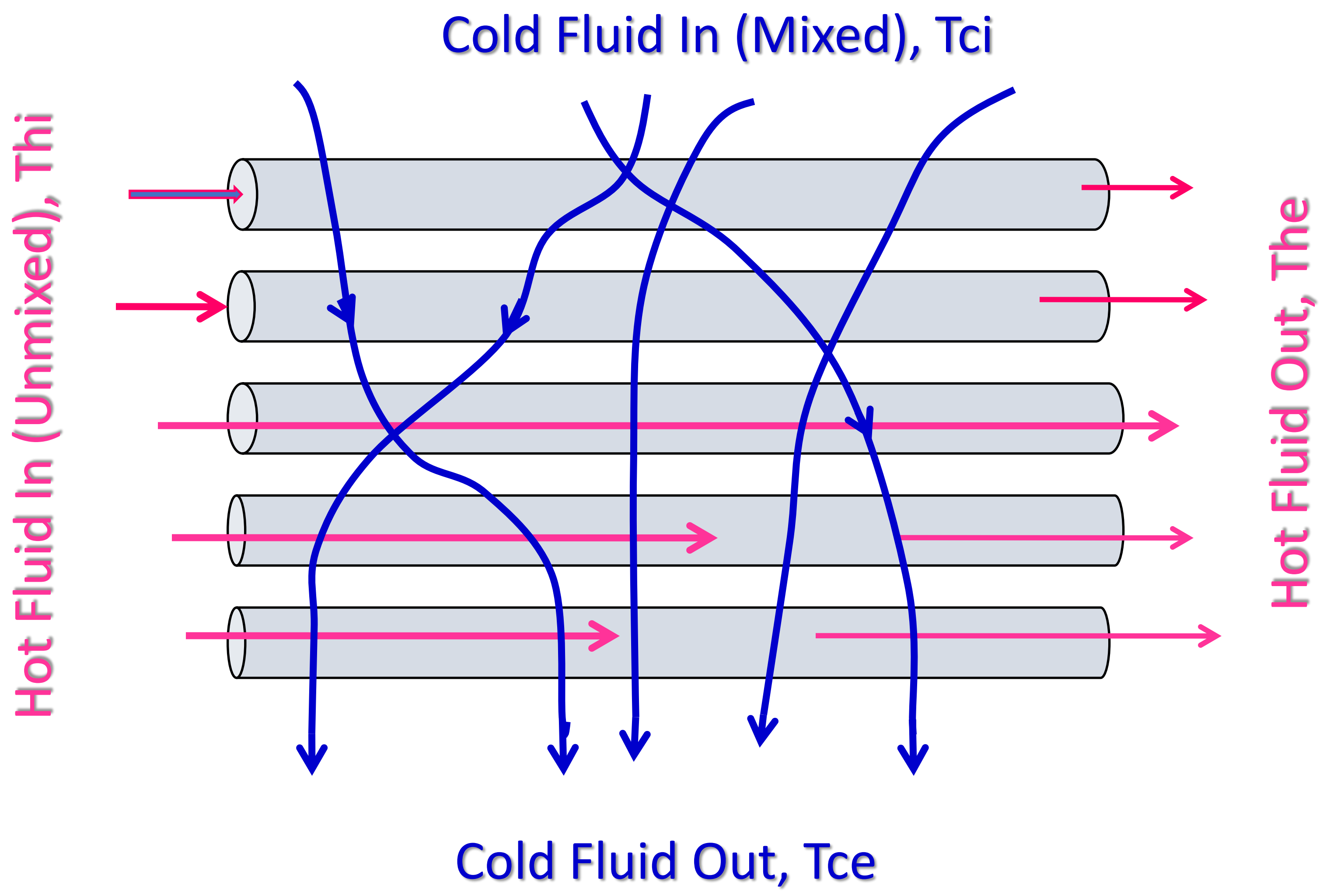
Tubular Heat Exchanger (Concentric Tubes)



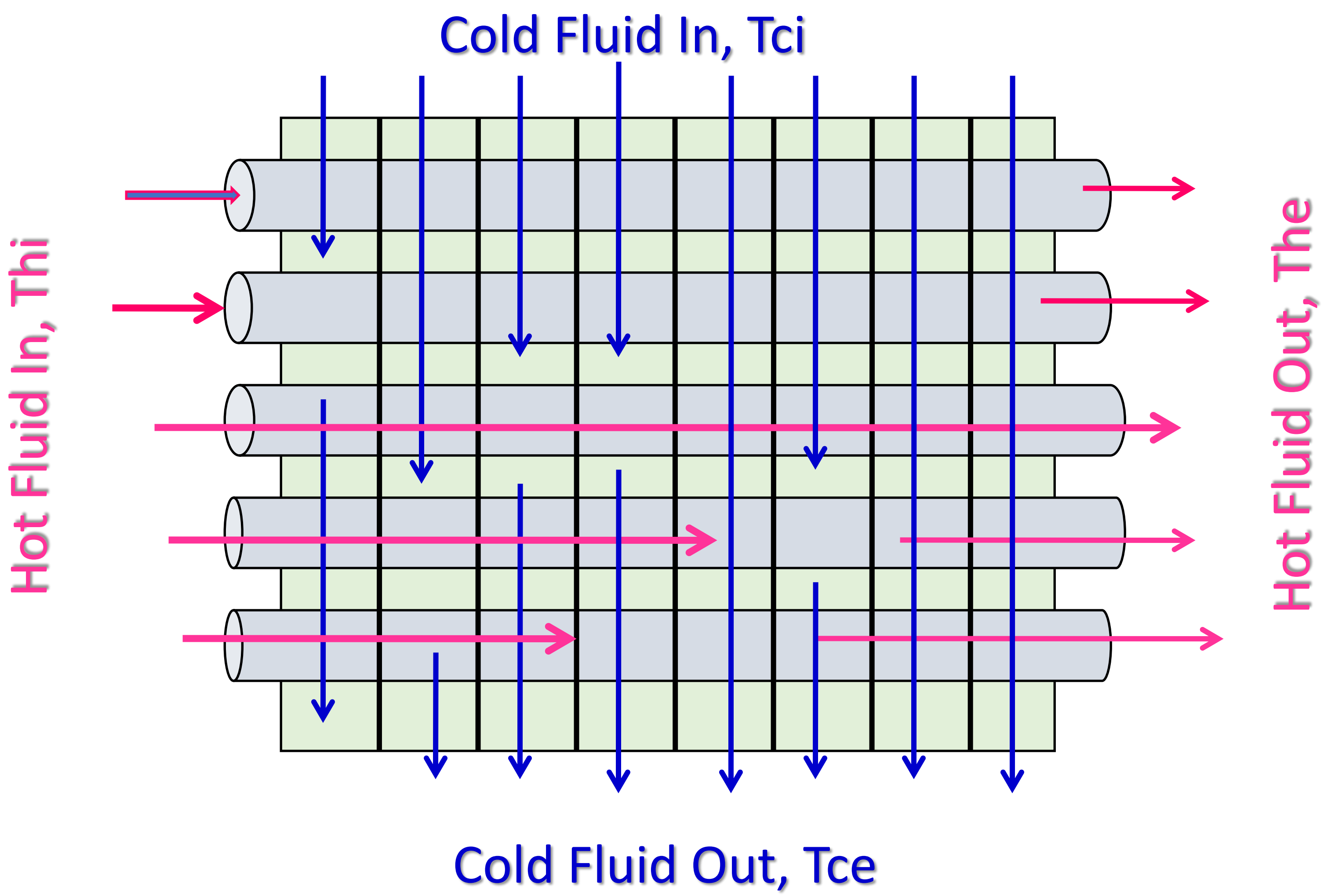
Counter Flow HE



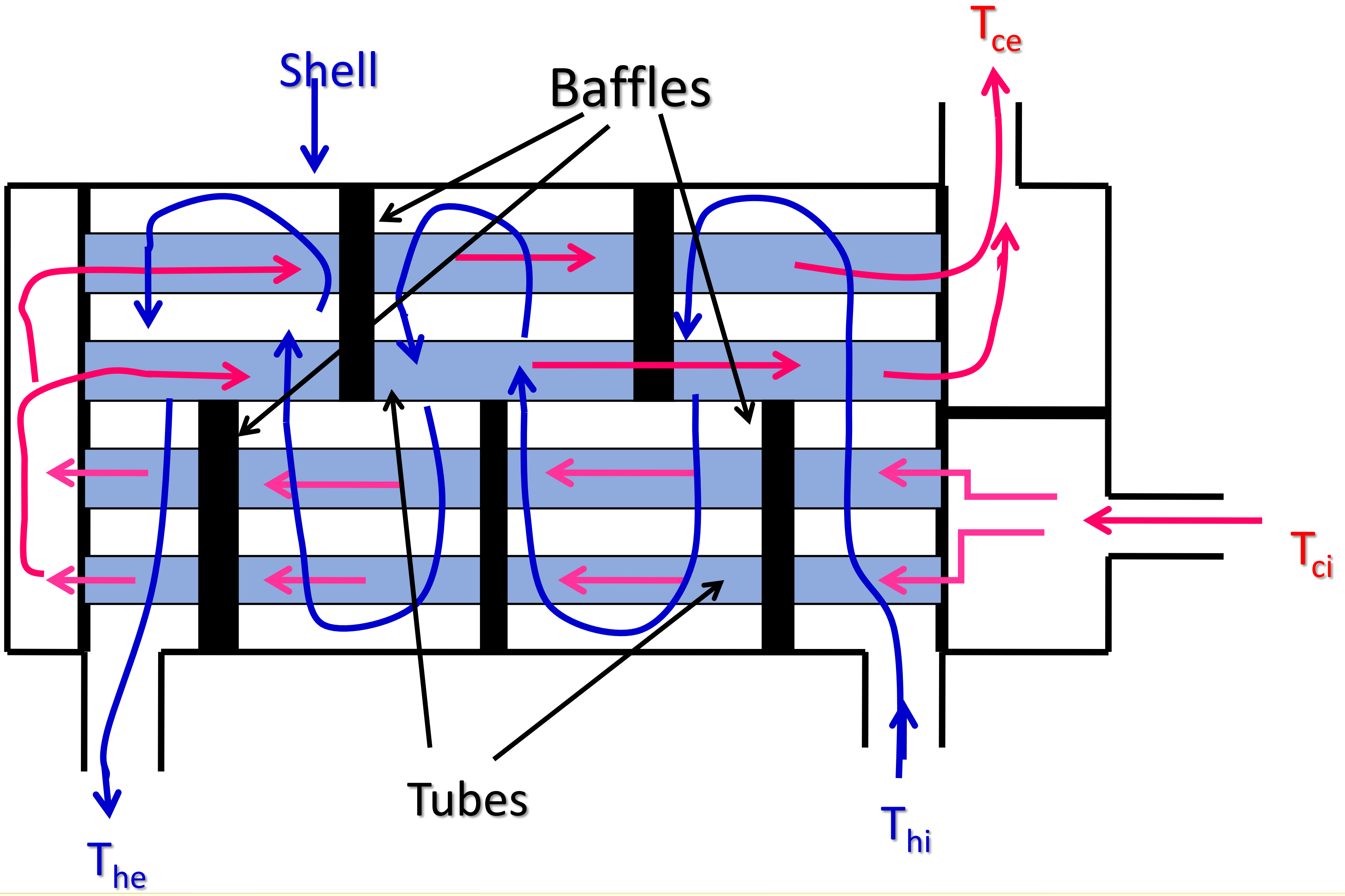
Cross Flow HE (One Fluid Mixed, One Unmixed):



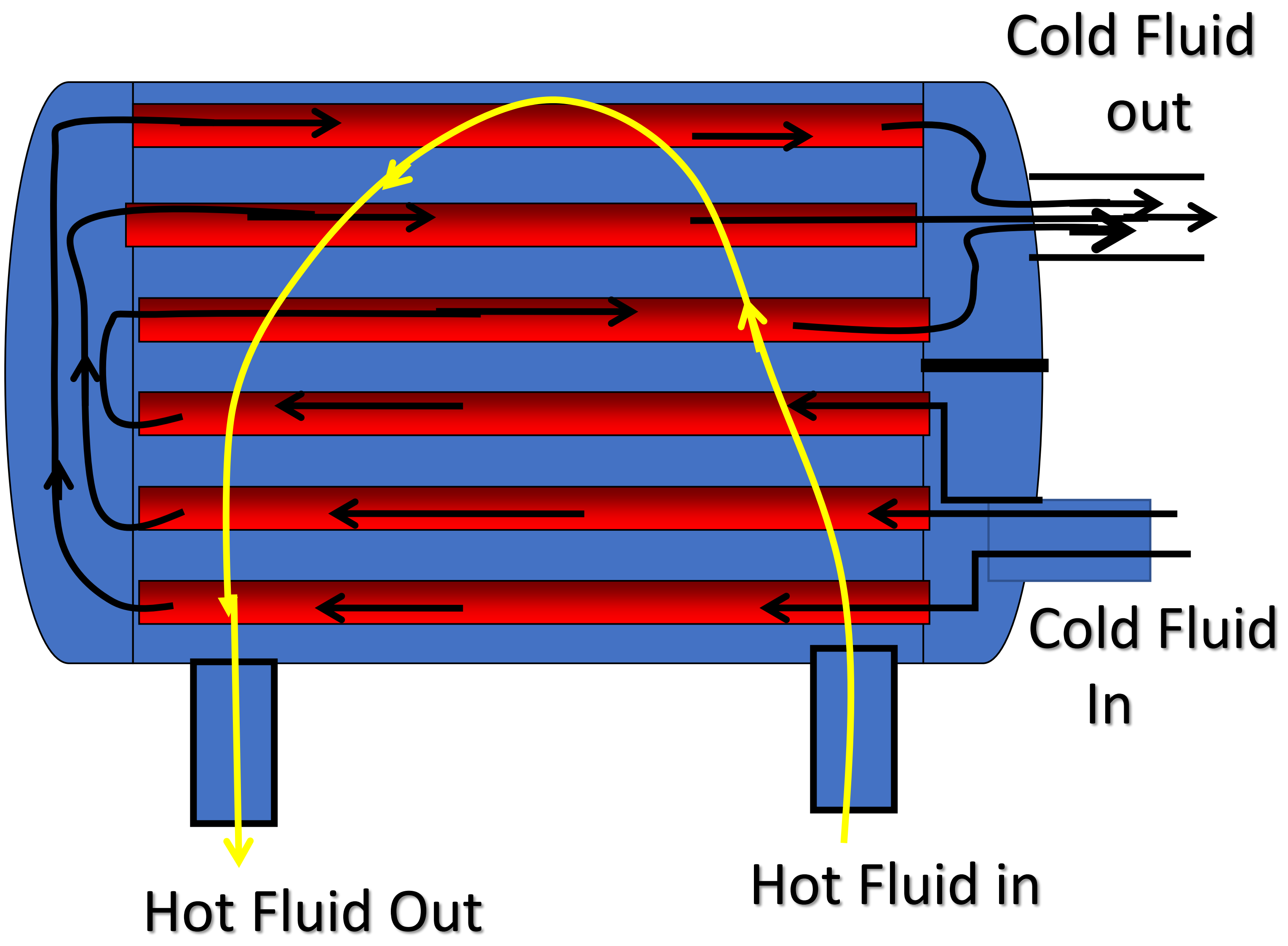
Cross Flow HE (Both Fluids Unmixed):



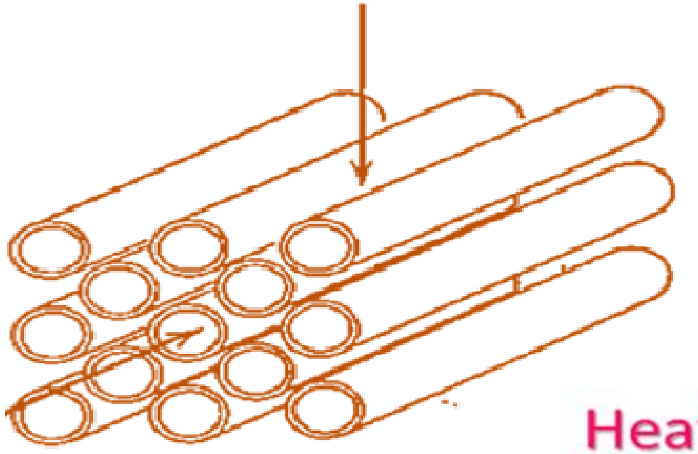
Direct Transfer Type HE (Shell-Tube Type)

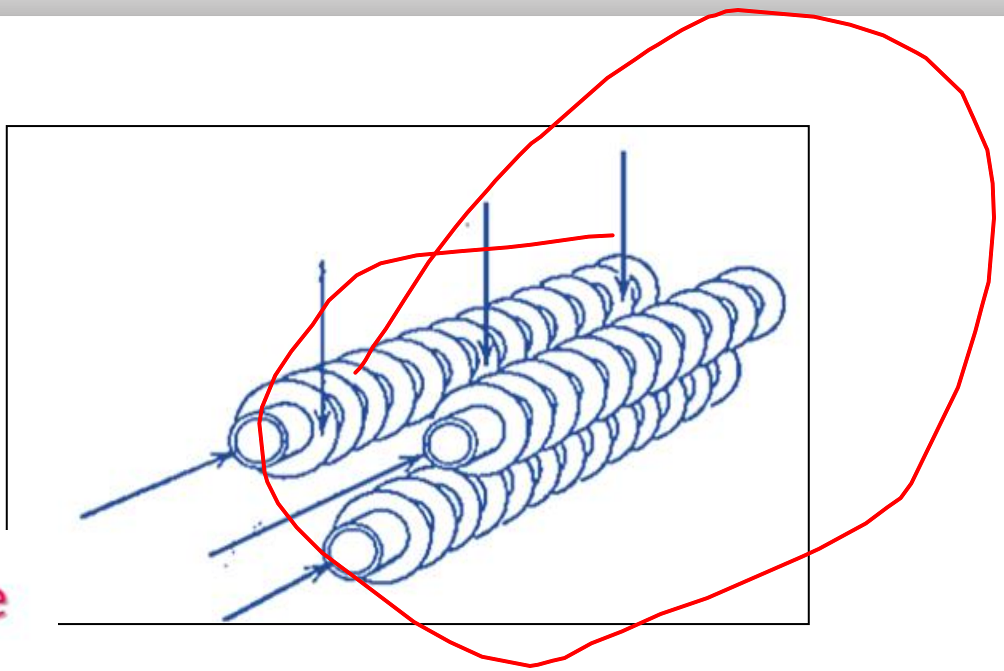


Heat Exchanger

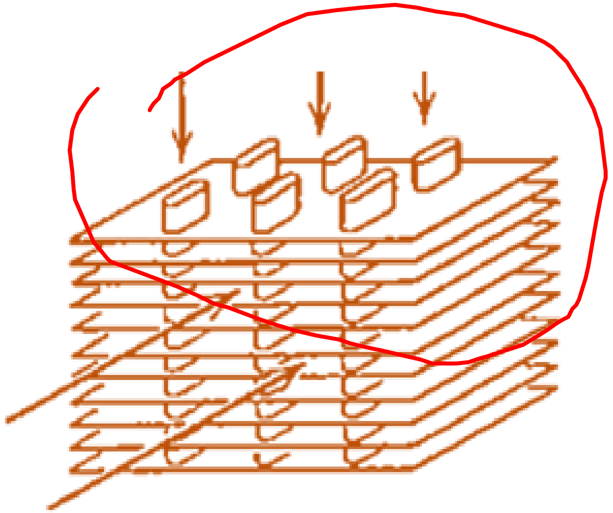


Compact Heat Exchangers

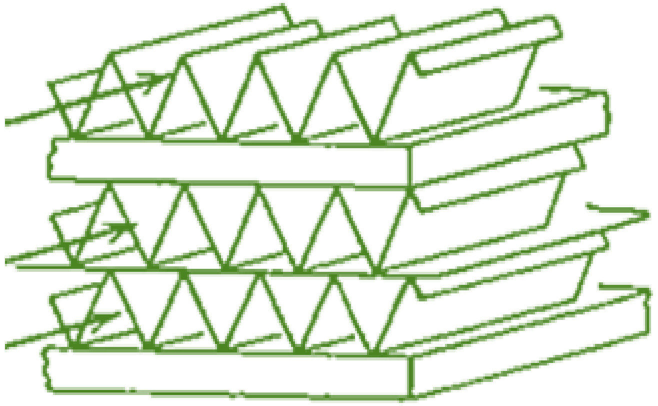




Heat Transfer Surface Area >$700 m^2/m^3$ on either or both sides



$D_h < 6mm$



Typical equipment consists of a bundle of parallel tube encased in a cylindrical shell

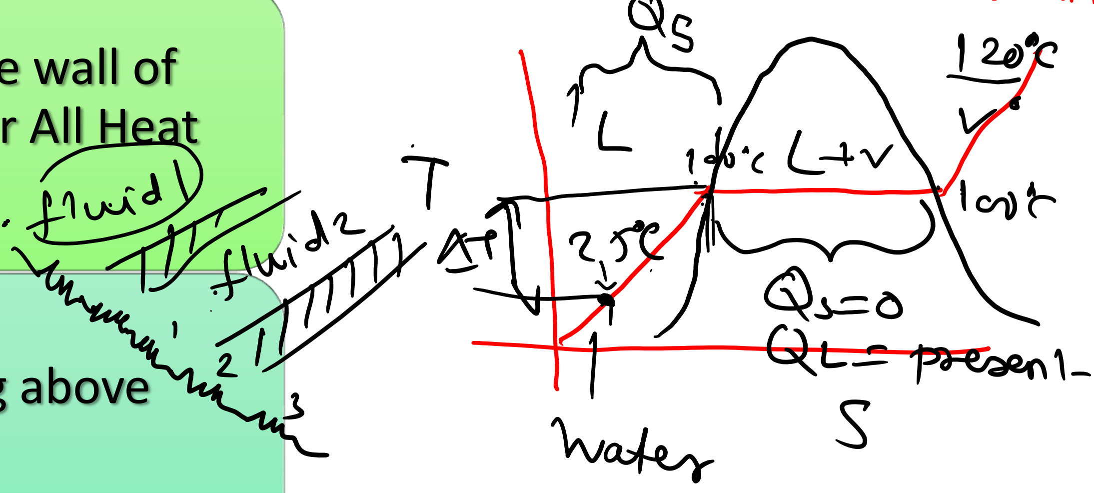
Some Important Aspects of HEs

Heat energy given by hot fluid = Energy gained by cold fluid $(m.Cp.\Delta T)_{hot\ fluid}$ = $(m.Cp.\Delta T)_{cold\ fluid}$

In direct transfer type HEs, transfer of energy takes place across the wall of metal and rate of heat flow can be estimated using the term ' Over All Heat Transfer' as: $Q = U.A.\Delta T$

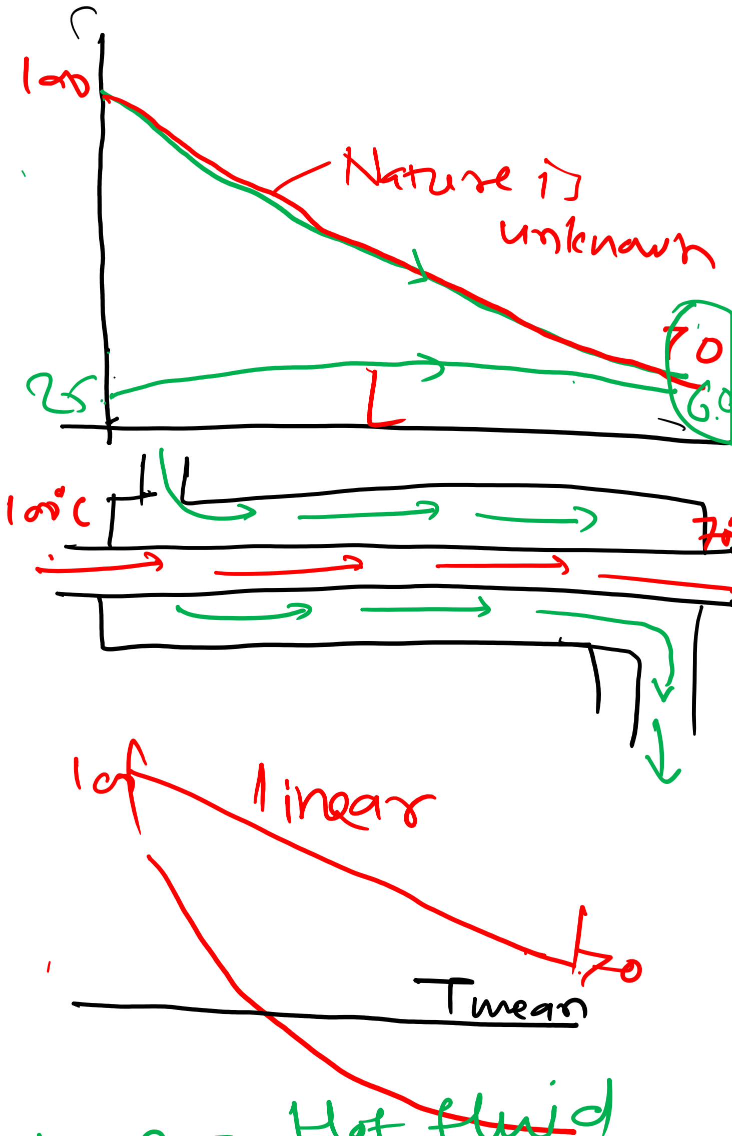
In HEs, ΔT varies across the length of HE, therefore, while applying above formula, some mean temp difference has to be used.

- Surfaces of HEs get coated with deposits with passage of time resulting in deterioration of performance. The effect of deposits/scales is represented by FOULING FACTOR, which has to be added to other thermal resistances for evaluation of over all heat transfer coefficient.



Representative Values of Fouling Factors

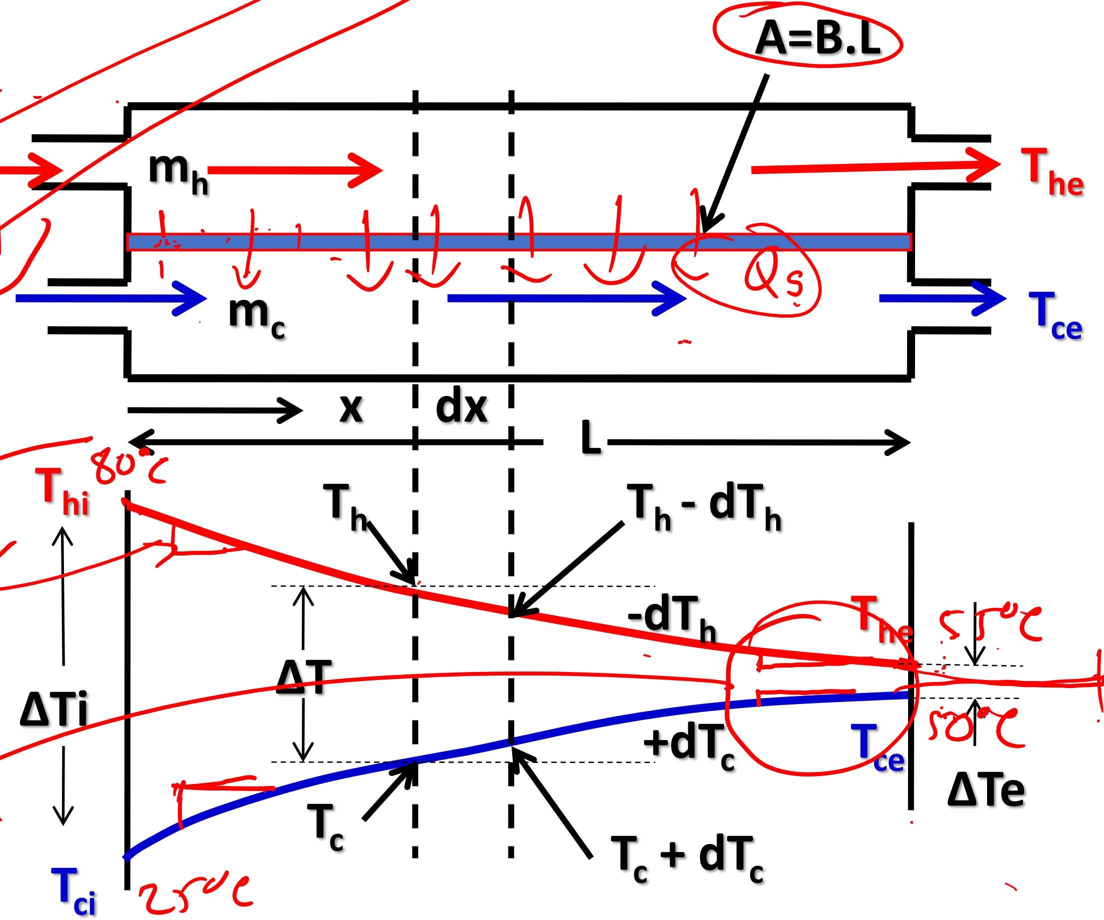
Fluid	Fouling Factor, m²-K/W
Seawater and treated boiler feedwater (<50°C)	0.0001
Seawater and treated boiler feedwater (>50°)	0.0002
River water (<50°C)	0.0002-0.001
Fuel oil	0.0009
Refrigerating liquids	0.0002
Steam	0.0001



LTM If exit temp = Hot fluid exit temp

Assumptions:

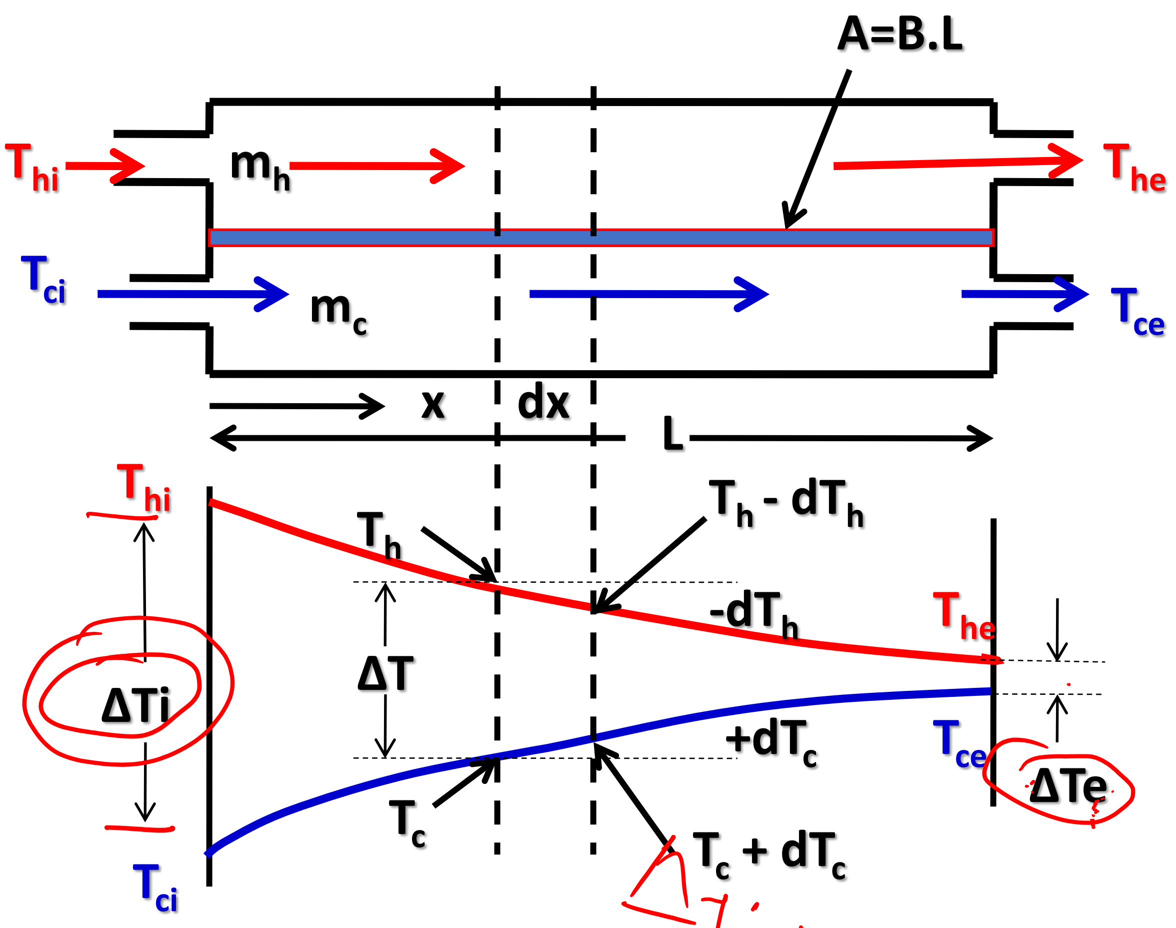
- Heat transfer takes place only between two fluids
- U is const through out
- C_p of fluids are const
- No temp gradient across the wall
- No change in KE & PE of the fluids



- Consider HE, in which heat is transferred across an area A of width B and length L.
- Let flow rates on hot and cold sides be m_h & m_c respectively

Analysis of Parallel Flow HE

Thi- hot fluid temp at inlet
The- hot fluid temp at exit
Tci- cold fluid temp at inlet
Tce- cold fluid temp at exit



From the Fig, temp diff at inlet is max Δti and min at exit ΔTe.

Consider an elemental area dA at distance x of length dx.

Let the temp at the beginning of elemental area be T_h and T_c and let the change in temps while they flow over area dA be dT_h and dT_c as shown in Fig.

Analysis of Parallel Flow HE

For steady state conditions,

Rate of Heat Transfer = Rate of change of Internal energy of the fluid

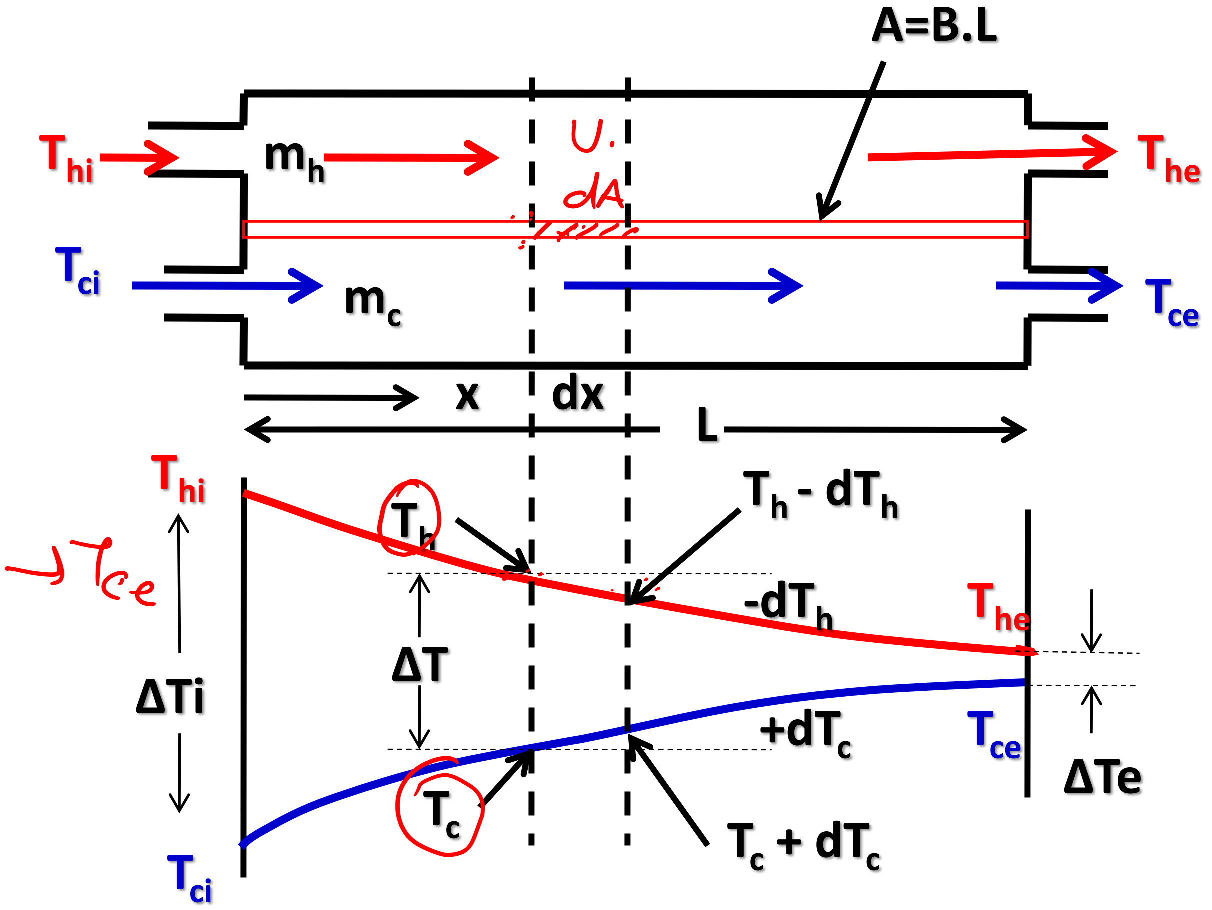
Therefore,

$Q = U.dA.\Delta T$

$= m_h.C_{ph}.(-dT_h)$

$= m_c.C_{pc}.(+dT_c)$……..(1);

where dA=B.dx



At the beginning of the elemental area dA, we can write;

$\Delta T = T_h - T_c$ …………..(2)

Differentiating (2), We have $d(\Delta T) = dT_h - dT_c$ …………..(3)

Substituting values of dT_h & dT_c from eqn..(1) in (3), we have;

$UA\,\Delta T = m C_p\, d(\Delta T)$

$$d(\Delta T) = \frac{U.dA.\Delta T}{-m_h.C_{ph}} - \frac{U.dA.\Delta T}{m_c.C_{pc}} \quad OR \quad \frac{d(\Delta T)}{\Delta T} = -\left(\frac{1}{m_h C_{ph}} + \frac{1}{m_c C_{pc}}\right).U.B.dx$$

dA

$$Integrating \int_{\Delta Ti}^{\Delta Te} \frac{d(\Delta T)}{\Delta T} = -\left(\frac{1}{m_h C_{ph}} + \frac{1}{m_c C_{pc}}\right).U.B \int_0^L dx \quad OR \quad [\ln \Delta T]_{\Delta Ti}^{\Delta Te} = -\left(\frac{1}{m_h C_{ph}} + \frac{1}{m_c C_{pc}}\right).U.B.L$$

$\int \frac{1}{x} dx = \ln x$

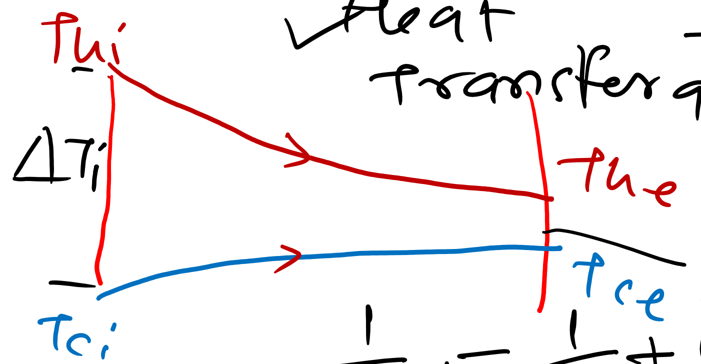
$\ln \Delta T_i - \ln \Delta T_e = \ln\left(\frac{\Delta T_i}{\Delta T_e}\right)$

$$OR \quad \ln \Delta Te - \ln \Delta Ti = -\left(\frac{1}{m_h C_{ph}} + \frac{1}{m_c C_{pc}}\right).U.A \qquad Or \; \ln\left(\frac{\Delta Te}{\Delta Ti}\right) = -\left(\frac{1}{m_h C_{ph}} + \frac{1}{m_c C_{pc}}\right)U.A \ldots\ldots\ldots(4)$$

Analysis of Parallel Flow HE

$Q = mC_p\Delta T \rightarrow$ Thermodyn. approach

Heat transfer approach $\rightarrow Q = U A \, LMTD$



Also, $Q = m_h C_{ph}(T_{hi} - T_{he}) = m_c C_{pc}(T_{ce} - T_{ci})$

$$\Rightarrow \frac{1}{m_h C_{ph}} = \frac{T_{hi} - T_{he}}{Q} \; \& \; \frac{1}{m_c C_{pc}} = \frac{T_{ce} - T_{ci}}{Q}$$

Substituting in eqn.......(4), We have;

$$\ln\left(\frac{\Delta Te}{\Delta Ti}\right) = -\left(\frac{T_{hi} - T_{he}}{Q} + \frac{T_{ce} - T_{ci}}{Q}\right) U.A$$

OR $\mathbf{Q = \dfrac{-U.A}{\ln\left(\dfrac{\Delta Te}{\Delta Ti}\right)}\left[(T_{hi} - T_{ci}) - (T_{he} - T_{ce})\right]}$

$$Q = \frac{U.A.(\Delta T_i - \Delta T_e)}{\ln\left(\frac{\Delta T_i}{\Delta T_e}\right)}$$

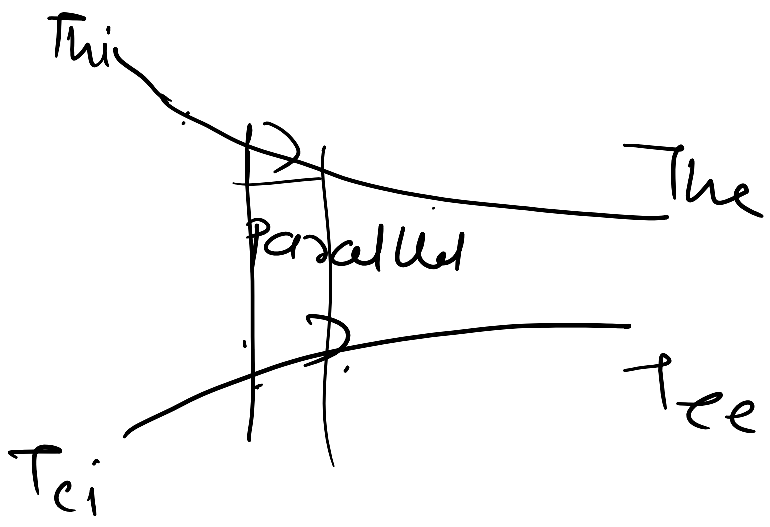
$$LMTD = \frac{\Delta T_i - \Delta T_e}{\ln\left(\frac{\Delta T_i}{\Delta T_e}\right)}$$

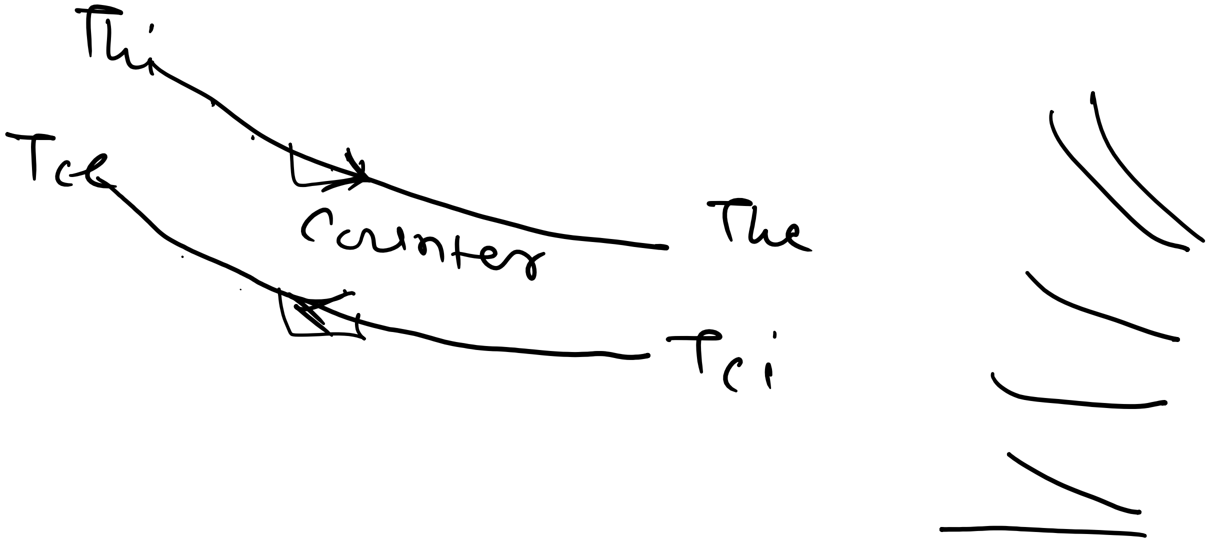
Analysis of Parallel Flow HE

$$Q = U.A.\frac{\Delta Ti - \Delta Te}{\ln\left(\frac{\Delta Ti}{\Delta Te}\right)}$$

$$Q = U.A.\Delta T_m$$

$Fromthis\ \ \exp \rightleftarrows ression\ \ Qcannowbecalculated$

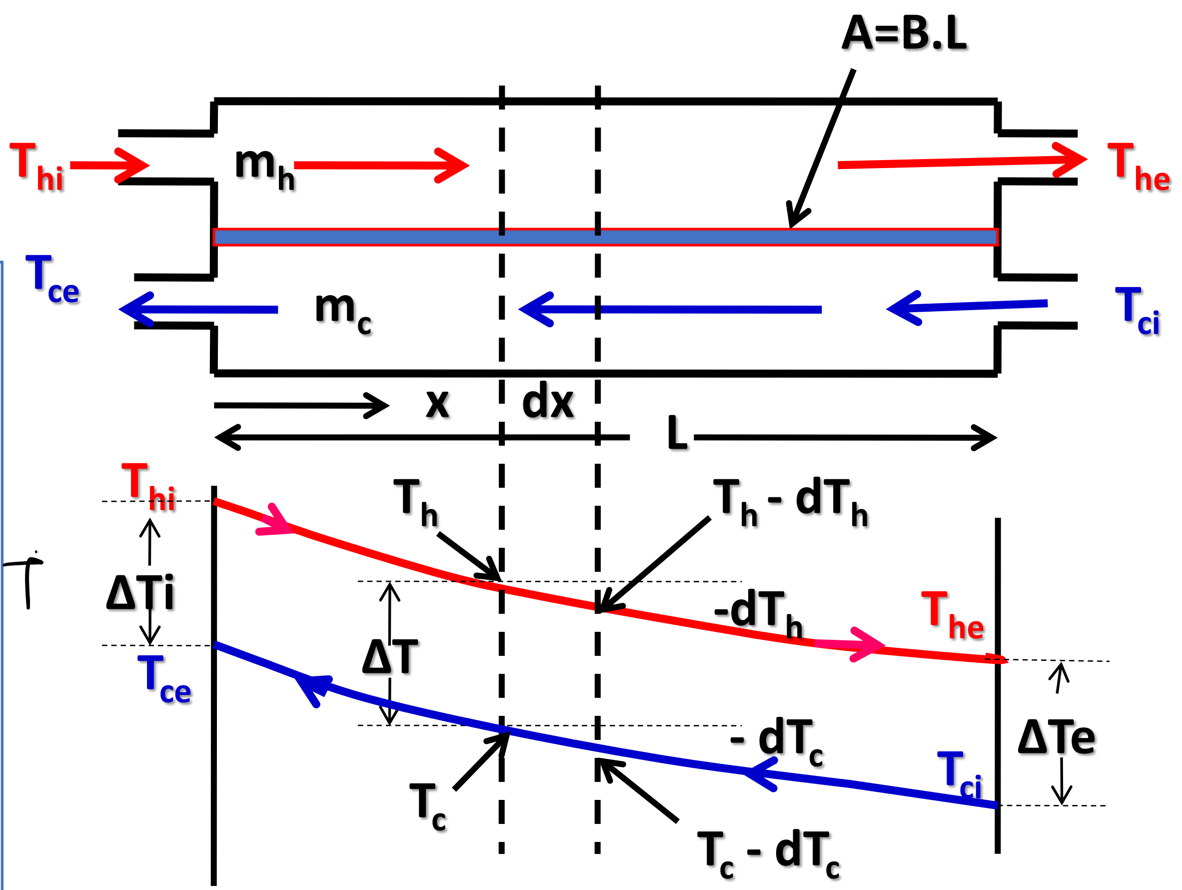




$$Comparingwehave;\ \Delta T_m = \frac{\Delta Ti - \Delta Te}{\ln\left(\frac{\Delta Ti}{\Delta Te}\right)}$$

Since ΔT_m contains log term, it is called Logarithmic Mean Temp Difference (LMTD)

Analysis of Counter Flow HE



Assumptions:

- Heat transfer takes place only between
- two fluids
- U is const through out
- C_p of fluids are const
- No temp gradient across the wall
- No change in KE & PE of the fluids

- Consider HE, in which heat is transferred across an area A of width B and length L.
- Let flow rates on hot and cold sides be m_h & m_c respectively

Analysis of Counter Flow HE

Consider HE, in which heat is transferred across an area A of width B and length L.

Let flow rates on hot and cold sides be m_h & m_c respectively

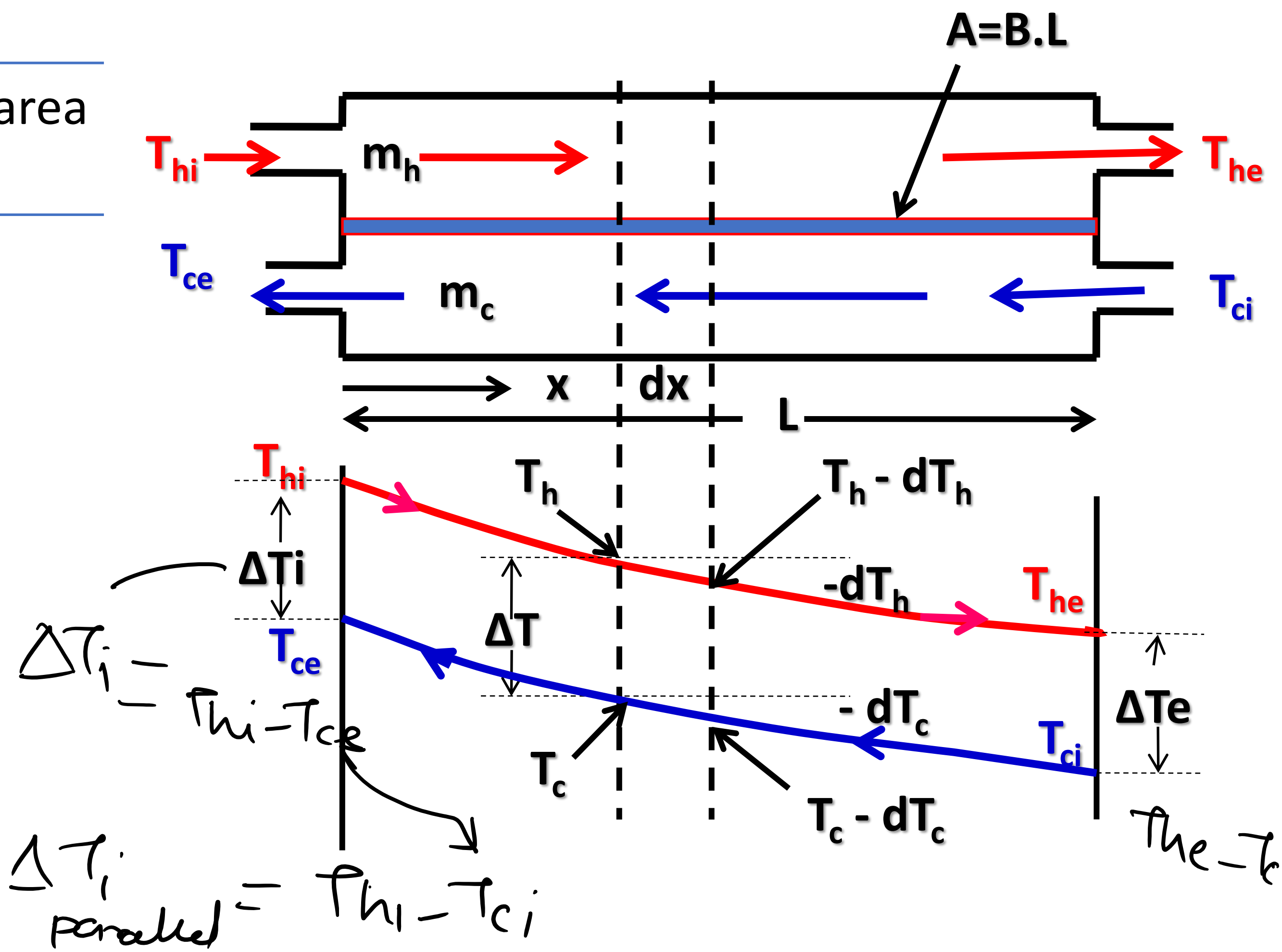
For steady state conditions

$$Q=U.dA.\Delta T$$

$$=m_h.C_{ph}.(-dT_h)$$

$$=m_c.C_{pc}.(-dT_c)\ldots\ldots(1);$$

where $dA=B.dx$



Analysis of Counter Flow HE

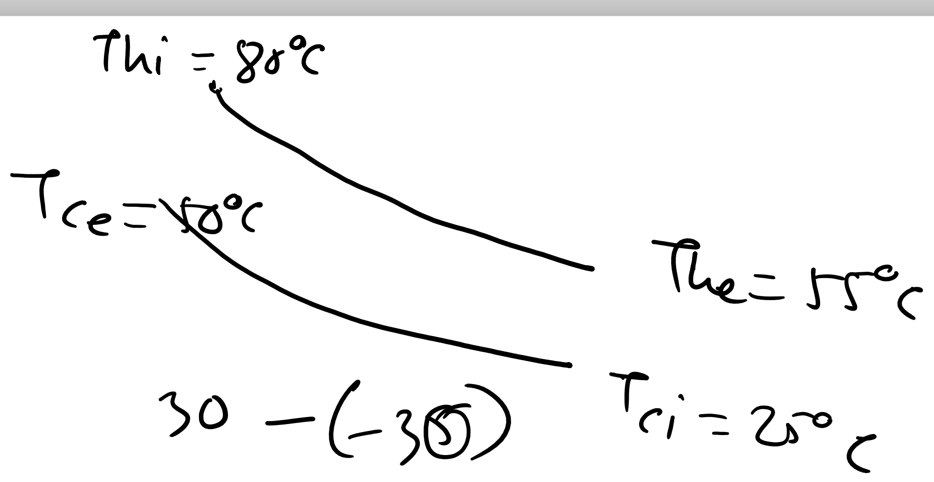
$$\frac{d(\Delta T)}{\Delta T} = -\left(\frac{1}{m_h C_{ph}} - \frac{1}{m_c C_{pc}}\right).U.B.dx$$

$$Integrating \int_{\Delta Ti}^{\Delta Te} \frac{d(\Delta T)}{\Delta T} = -\left(\frac{1}{m_h C_{ph}} - \frac{1}{m_c C_{pc}}\right).U.B\int_0^L dx$$

$$OR \quad [\ln \Delta T]_{\Delta Ti}^{\Delta Te} = -\left(\frac{1}{m_h C_{ph}} - \frac{1}{m_c C_{pc}}\right).U.B.L$$

$$\ln\left(\frac{\Delta Te}{\Delta Ti}\right) = -\left(\frac{1}{m_h C_{ph}} - \frac{1}{m_c C_{pc}}\right).U.A\ldots\ldots(4)$$

Analysis of Counter Flow HE



$$Also,\ Q = m_h C_{ph}(T_{hi} - T_{he}) = m_c C_{pc}(T_{ce} - T_{ci})$$

$$\Rightarrow \frac{1}{m_h C_{ph}} = \frac{T_{hi} - T_{he}}{Q} \ \&\ \frac{1}{m_c C_{pc}} = \frac{T_{ce} - T_{ci}}{Q}$$

$Substituting in eqn \ldots\ldots (4),\ We\ have;$

$$\ln\left(\frac{\Delta T_i}{\Delta T_e}\right) = \left(\frac{T_{hi} - T_{he}}{Q} - \frac{T_{ce} - T_{ci}}{Q}\right) U.A$$

$$OR \quad Q = \frac{U.A}{\ln\left(\frac{\Delta T_i}{\Delta T_e}\right)}[T_{hi} - T_{he} - T_{ce} + T_{ci}]$$

30 − (−30)

30 + 30

25 − 55

−ve

Tci − The

$$Q = \frac{U \cdot A \cdot (\Delta T_i + \Delta T_e)}{\ln\left(\frac{\Delta T_i}{\Delta T_e}\right)}$$

Analysis of Counter Flow HE

$$Q = \frac{U.A}{\ln\left(\frac{\Delta Ti}{\Delta Te}\right)}[(T_{hi} - T_{ce}) - (T_{he} - T_{ci})]$$

$$Q = U.A.\frac{\Delta Ti - \Delta Te}{\ln\left(\frac{\Delta Ti}{\Delta Te}\right)} = U.A.\Delta T_m$$

$$Comparing we have;\ \Delta T_m = \frac{\Delta Ti - \Delta Te}{\ln\left(\frac{\Delta Ti}{\Delta Te}\right)}$$

Parallel Flow	Counter Flow
$\Delta Ti = T_{hi} - T_{ci}$	$\Delta Ti = T_{hi} - T_{ce}$
$\Delta Te = T_{he} - T_{ce}$	$\Delta Te = T_{he} - T_{ci}$

Q now can be calculated from the expression:

Q=U.A.ΔT

Counter Flow HEs

$$LMTD = \frac{\Delta T_i - \Delta T_e}{\ln \frac{\Delta T_i}{\Delta T_e}} = \frac{0}{0}$$

$$Q = UA\,LMTD$$

A special case of Counter Flow HE occurs when the Capacity Rates on the two sides are equal:

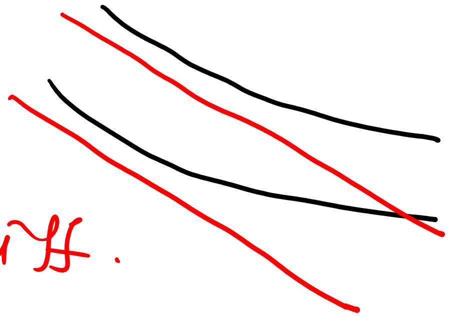
$m_h C_{ph} = m_c C_{pc} \rightarrow (T_{hi} - T_{ce}) = (T_{he} - T_{ci}) \rightarrow$ **ΔTi = ΔTe**

$Q_1 = m_c C_{pc} \Delta T_c = m_h C_{ph} \Delta T_h$

$(T_{ce} - T_{ci})$ $\Delta T_c = \Delta T_h$ $= T_{hi} - T_{he}$

$$Hence\ \Delta T_m = \frac{\Delta Ti - \Delta Te}{\ln \frac{\Delta Ti}{\Delta Te}}\ will\ become\ In \rightleftarrows \det \rightleftarrows er \rightleftarrows \min \rightleftarrows \rightleftarrows ate$$

indeterminate

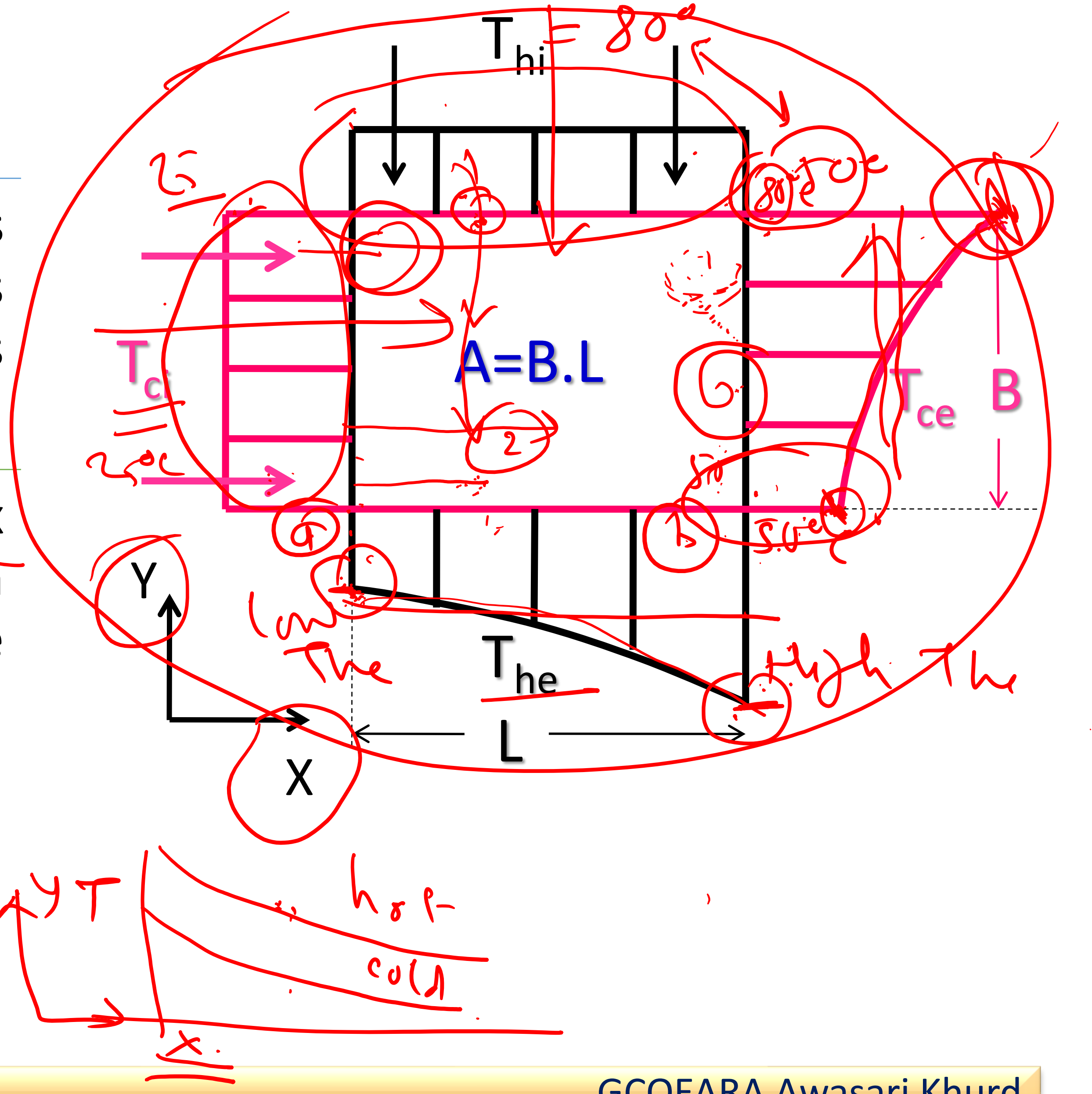


This can be solved by applying L' hospitals rule and it can be shown that ΔTm = ΔTe = ΔTi → arithmetic temp. diff.

Cross Flow HE

In both parallel and counter flow HEs, temps on both sides vary only along the length of HEs and are function of single variable, say x. This is not so in case of cross flow HE.

It is obvious that T_h & T_c are now function of x and y and exit temp profiles are not uniform Determination of LMTD involves double integration and becomes complicated

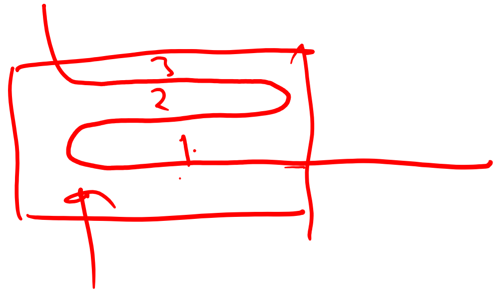


Cross Flow HE

Study and development of relations for cross flow and for many other types multi-pass flow arrangements was carried out by Bowman, Mueller and Nagle.

Some of these are:

- - Both fluids unmixed
- - Both fluids mixed
- - One fluid mixed, one unmixed
- - One Shell & Two tube passes (and multiples of 2)
- - Two Shell passes and multiple tube passes



Under these conditions, heat transfer rate is calculated as:

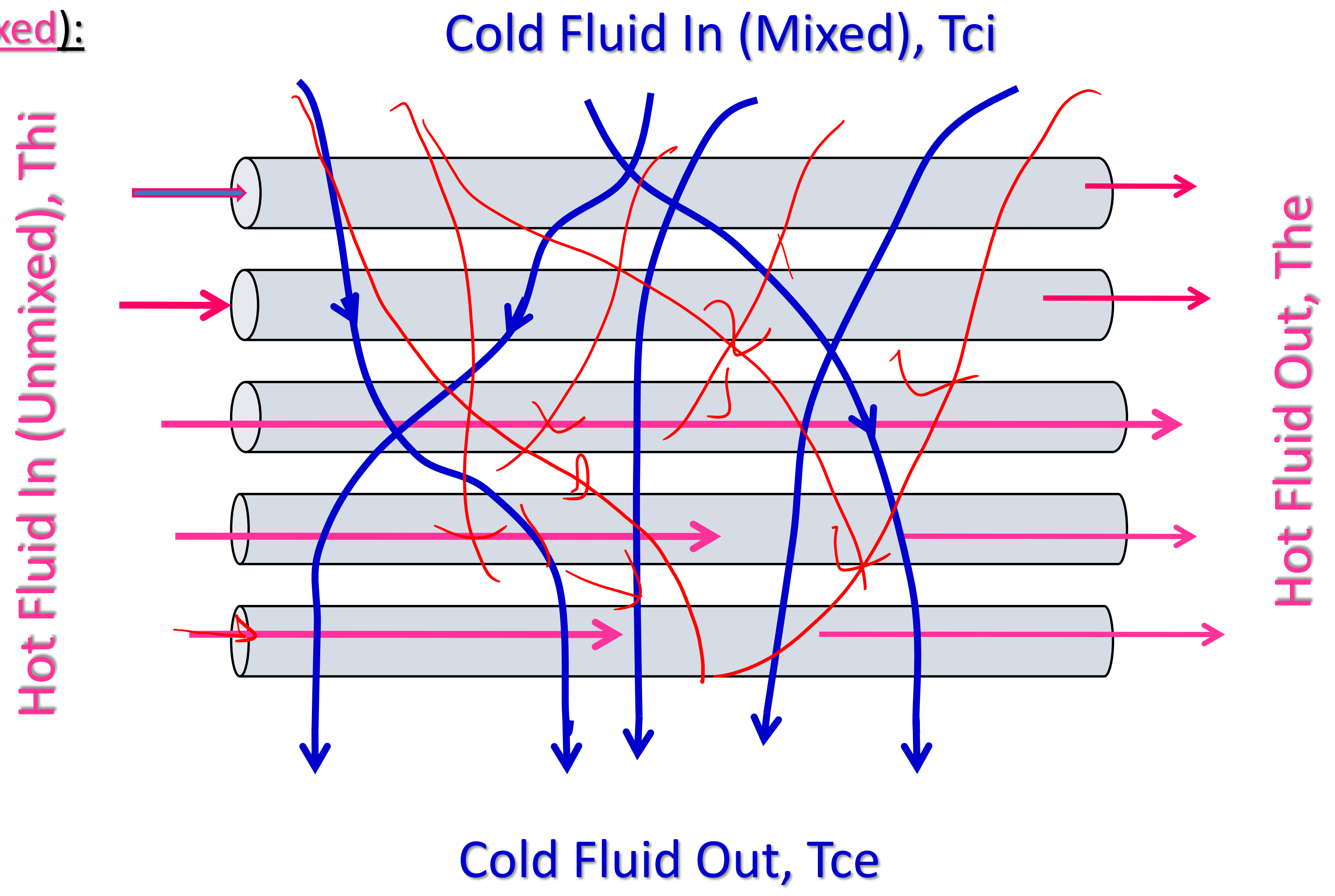
- $Q = U.A.F.(\Delta T_m)_{counter\ flow}$, where F is correction factor, which graphically determined with the help of two parameters R and S

$Q = U A \Delta T \times f$

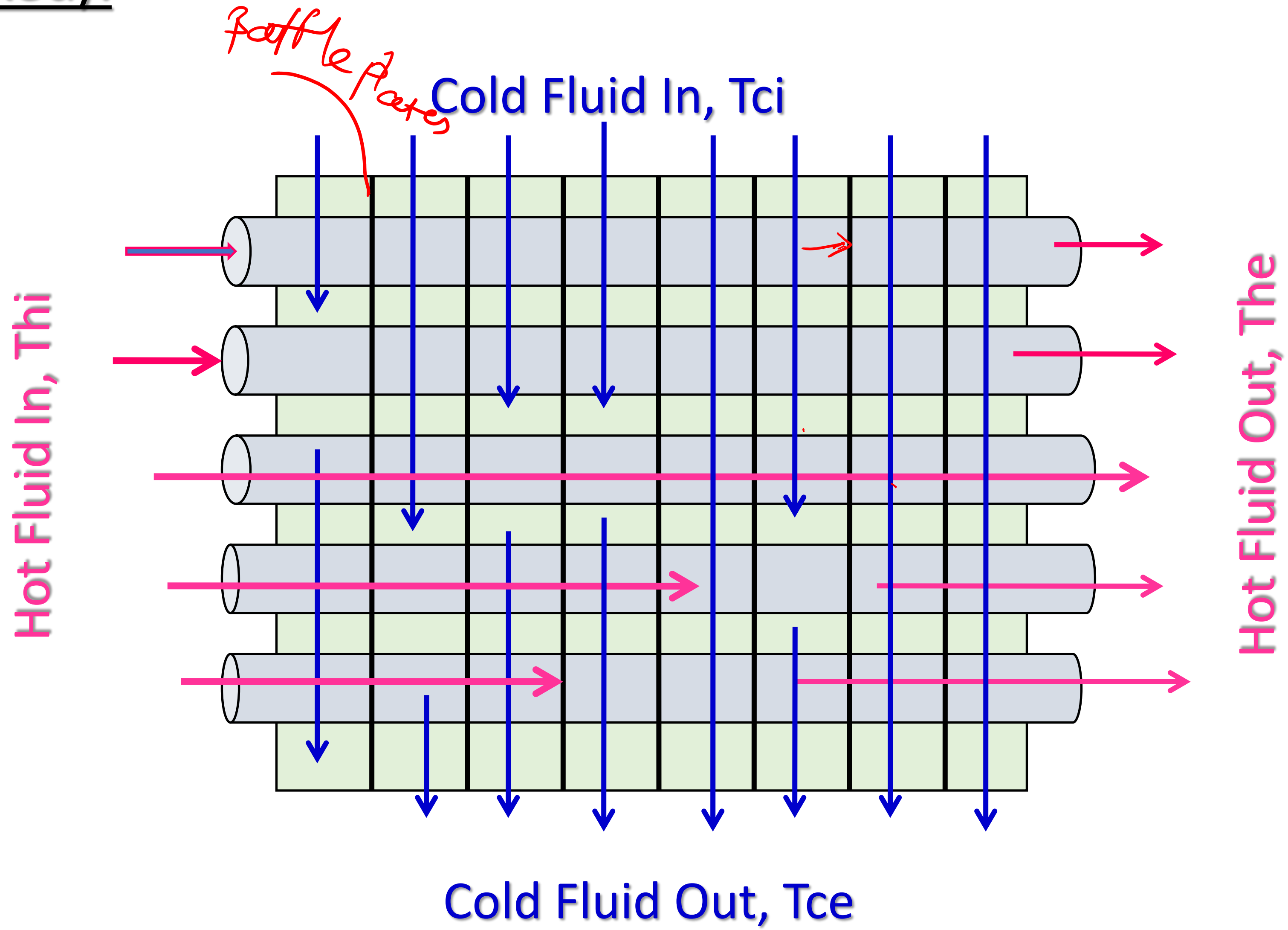
f → correction factor

Direct Transfer Type Heat Exchanger

Cross Flow HE (One Fluid Mixed, One Unmixed):



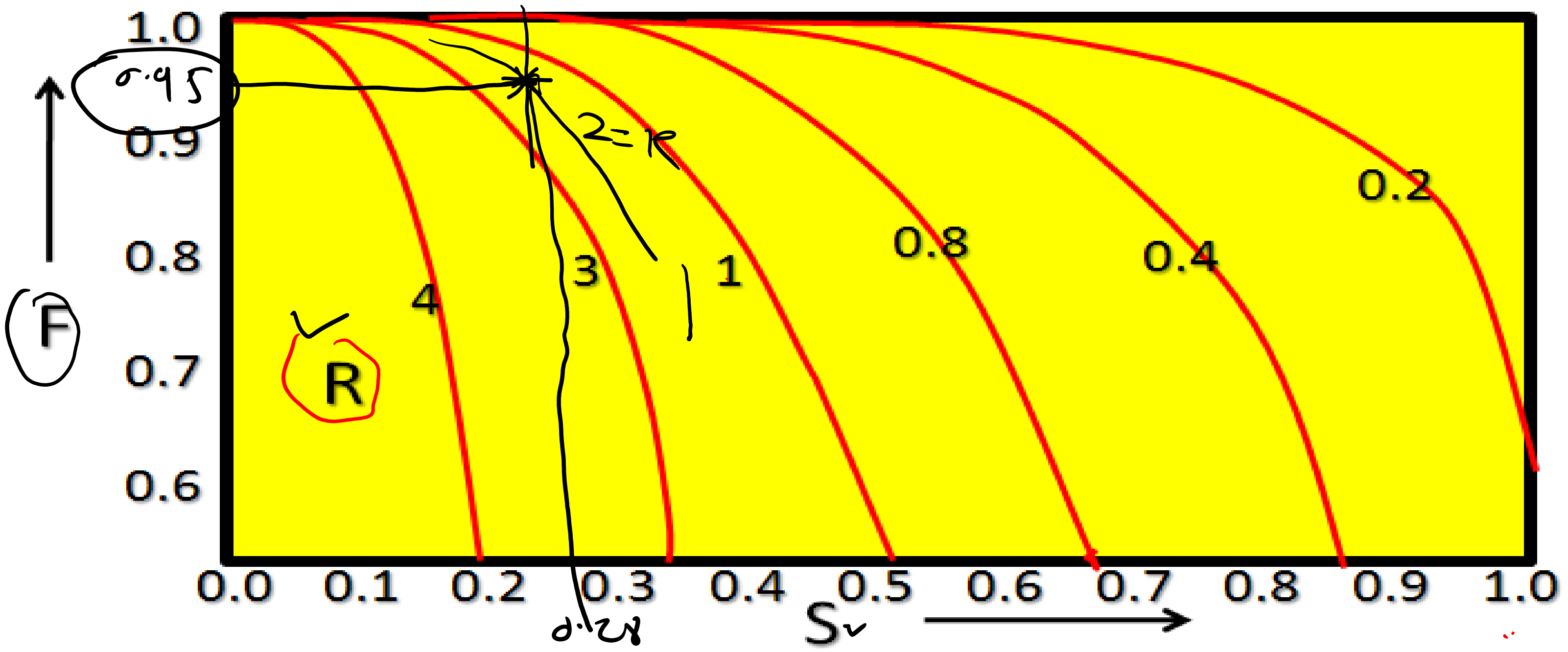
Cross Flow HE (Both Fluids Unmixed):



Cross Flow HE

$$R = \frac{T_{1i} - T_{1e}}{T_{2e} - T_{2i}} \; ; \quad S = \frac{T_{2e} - T_{2i}}{T_{1i} - T_{2i}}$$

Subscript 1 must be assigned to MIXED fluid, in case one fluid is mixed and other is UNMIXED. In other cases, any subscript can be assigned to any fluid.



$R = \frac{80-40}{30-10} = 2$

$S = \frac{30-10}{80-10} = \frac{20}{70} = 0.286$

$T_{1i} = 80$, $T_{2i} = 10$, $T_{2e} = 30$, $T_{1e} = 40$

F = based on R & S

Special Cases of HEs

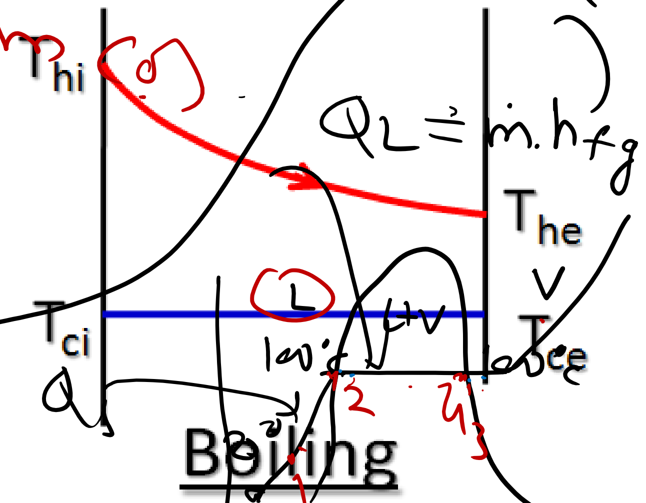
Two special cases occur, when the product of the flow rate and specific heat (mC_p) is INFINITE either on hot or cold side.

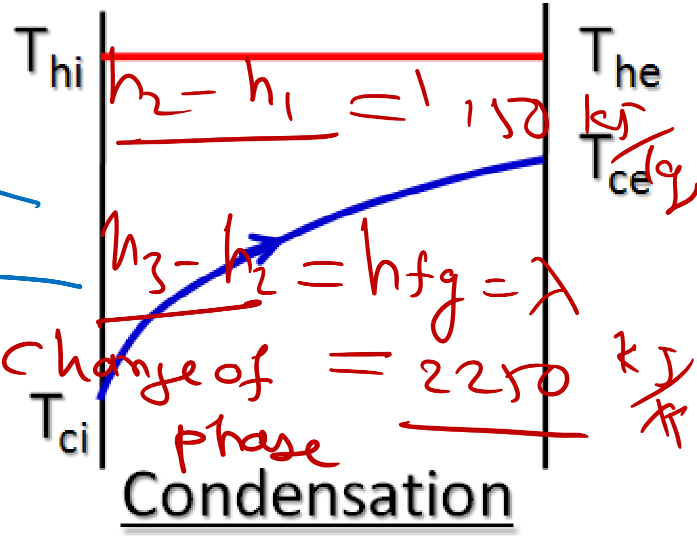
This happens when one of the fluids changes phase (Boiling and Condensation) and Q is calculated as $Q=m\lambda$.

$$m_h C_{ph}(T_{hi} - T_{he}) = m_c C_{pc}(T_{ce} - T_{ci})$$

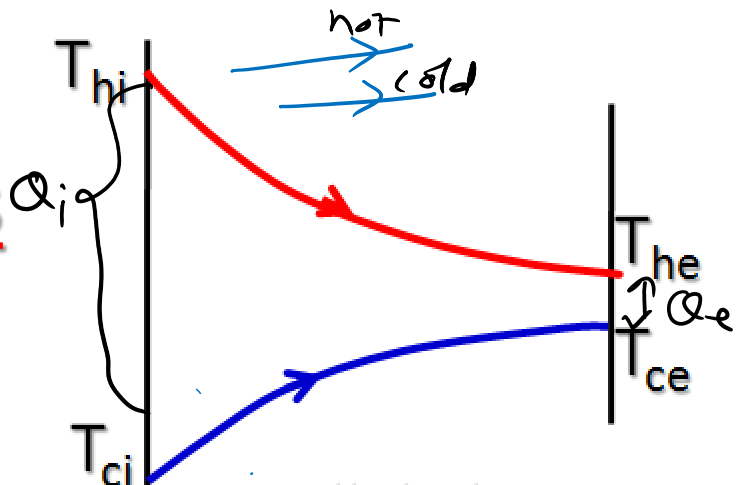
For Condensation: $T_{hi} - T_{he} = 0$ as $T_{hi} = T_{he}$

Since LHS = RHS and $T_{hi} - T_{he} = 0$, Therefore $m_h C_{ph} = \infty$

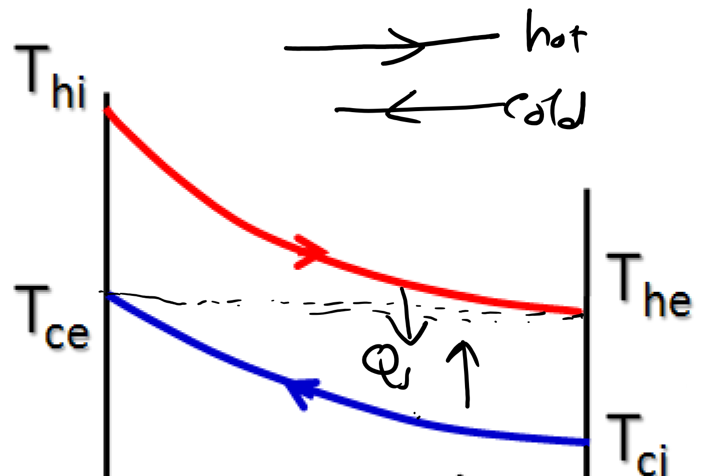




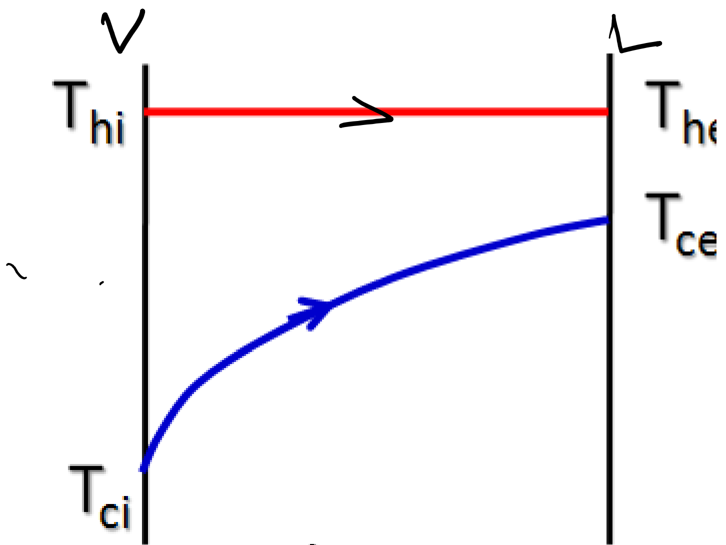
Temperature Profiles



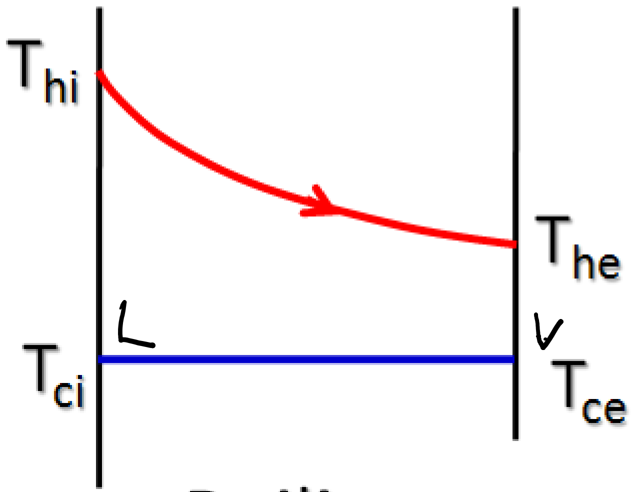
Parallel Flow



Counter Flow



Condensation



Boiling

NTU-Effectiveness Method

- While designing and testing, two types of problems are required to be tackled:
- For the two given fluids, mass flow rates and inlet & exit temps are specified, and size of HE is required to designed for specified performance.
- For given HE, only inlet temps and mass flow rates of the two fluids are specified and exit temps are required to be found out. This type of problem is basically evaluation of performance of a given heat exchanger.
- First kind of problems can be solved by LMTD method but second type of problems can not be solved by LMTD method.
- To solve the second type of problems, NTU-Effectiveness is used. However, first type of problems can also be solved by this method.

NTU-Effectiveness Method

- For this method, 3 parameters will be defined:

1. **Capacity Ratio (C):** It is defined as the ratio of heat capacities of the two fluids and is given as:

$$C = \frac{(mC_p)_{small}}{(mC_p)_{large}}; \quad 0 \le C \le 1$$

2. **Number of (Heat) Transfer Units (NTU):**

$$NTU = \frac{U.A}{(mC_p)_{Small}}$$

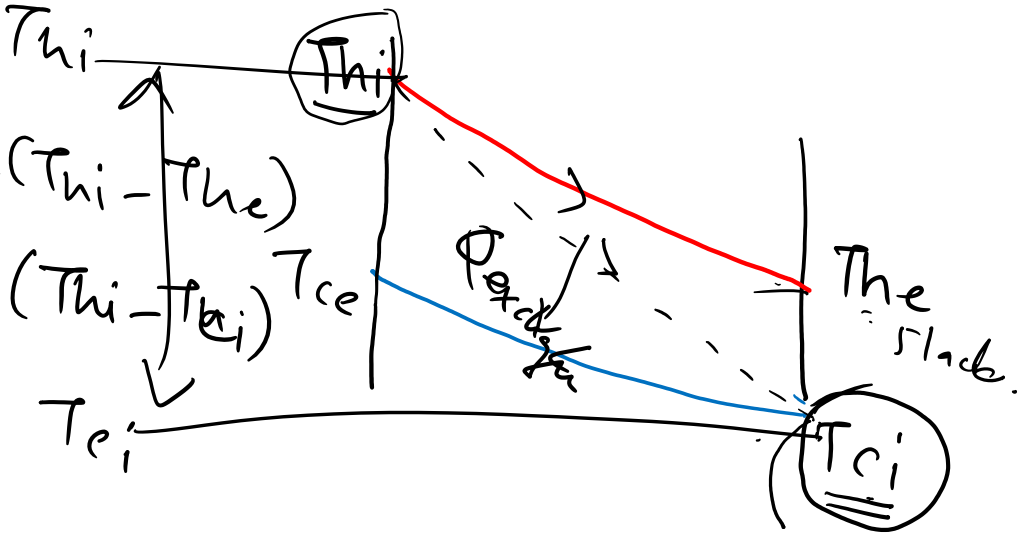
- For specified value of U/mC_p, NTU is the measure of actual heat transfer area A or size of the HE.

NTU-Effectiveness Method

3. Effectiveness (ε):

$Q_{act} = \dot{m}_h C_{ph} (T_{hi} - T_{he})$

$Q_{max} = \dot{m} C_{p\,small} (T_{hi} - T_{ci})$



$$Effectiveness(\varepsilon) = \frac{ActualHeatTransferRate(inHE)}{MaxPossibleHeatTransferRate(inHE)}$$

- Max heat transfer will occur when one of the fluids undergoes max temp change.

for fin ε > 1

- When Q =$m_h C_{ph}(T_{hi} - T_{he}) = m_c C_{pc}(T_{ce} - T_{ci})$, obviously the fluid having smaller heat capacity will only undergo max temp in HE.

for Hx ε < 1

$$Hence\ \varepsilon = \frac{(\dot{m}C_p\Delta T)_{hot\ or\ cold}}{(\dot{m}C_p)_{small}(T_{hi} - T_{ci})}$$

0 < ε < 1

NTU-Effectiveness Method

3. Effectiveness (ε):

$$\varepsilon = \frac{(mC_p\Delta T)_{hot\ or\ cold}}{(mC_p)_{small}(T_{hi} - T_{ci})}$$

- For $(mC_p)_h < (mC_p)_c$:

$$\varepsilon = \frac{(mC_p)_h(T_{hi} - T_{he})}{(mC_p)_h(T_{hi} - T_{ci})} = \frac{T_{hi} - T_{he}}{T_{hi} - T_{ci}}$$

Hot fluid in Numerator

- For $(mC_p)_c < (mC_p)_h$:

$$\varepsilon = \frac{(mC_p)_c(T_{ce} - T_{ci})}{(mC_p)_c(T_{hi} - T_{ci})} = \frac{T_{ce} - T_{ci}}{T_{hi} - T_{ci}}$$

cold fluid in numerator

NTU-Effectiveness Method For Parallel Flow HE

- Let $(mC_p)_h < (mC_p)_c$

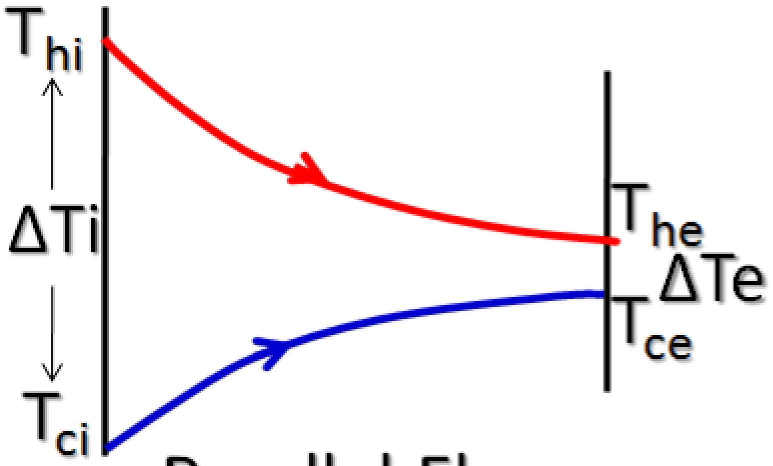
$$Hence\ C = \frac{(mC_p)_h}{(mC_p)_c}$$

m Cp small / mCp large

- Since $(mC_p)_h(T_{hi}-T_{he})=(mC_p)_c(T_{ce}-T_{ci})$

$$\therefore \frac{(mC_p)_h}{(mC_p)_c} = \frac{T_{ce}-T_{ci}}{T_{hi}-T_{he}} = C \ldots\ldots.(1)$$

$$And \quad \varepsilon = \frac{(mC_p)_h(T_{hi}-T_{he})}{(mC_p)_h(T_{hi}-T_{ci})} = \frac{T_{hi}-T_{he}}{T_{hi}-T_{ci}} \ldots\ldots.(2)$$



Parallel Flow

NTU-Effectiveness Method for Parallel Flow HE

Let us obtain T_{ci} & T_{ce} in terms of T_{hi} & T_{he} :

$$\left(T_{hi} - T_{ci}\right) = \frac{T_{hi}}{\varepsilon} - \frac{T_{he}}{\varepsilon}$$

From eqn ...(2), we have,

$$\Rightarrow T_{ci} = T_{hi} - \frac{T_{hi}}{\varepsilon} + \frac{T_{he}}{\varepsilon} \ldots\ldots\ldots(3)$$

And from eqn...(1), we have,

$$T_{ce} - T_{ci} = C.T_{hi} - C.T_{he}$$

$$\therefore \quad T_{ce} = T_{ci} + C.T_{hi} - C.T_{he}$$

Putting value of T_{ci} from eqn...(3)

$$T_{ce} = T_{hi} - \frac{T_{hi}}{\varepsilon} + \frac{T_{he}}{\varepsilon} + C.T_{hi} - C.T_{he} \ldots\ldots(4)$$

NTU-Effectiveness Method for Parallel Flow HE

$Here \rightleftarrows \quad \Delta T_i = T_{hi} - T_{ci}$

$$\Delta Ti = T_{hi} - (T_{hi} - \frac{T_{hi}}{\varepsilon} + \frac{T_{he}}{\varepsilon})$$

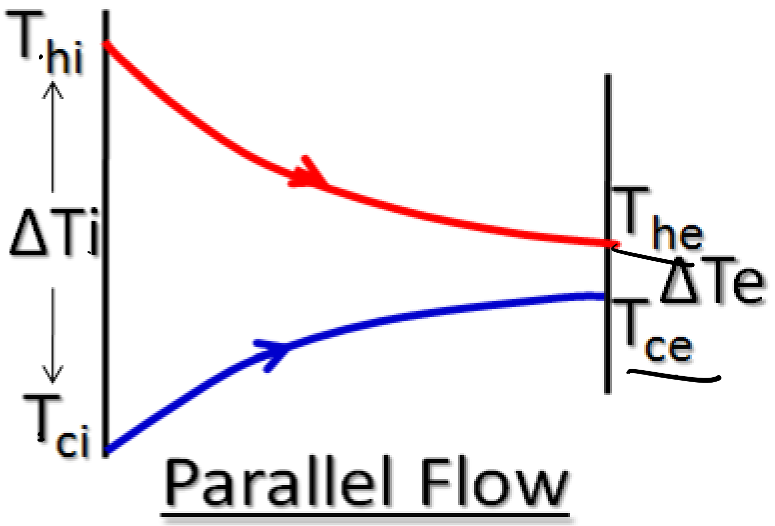
$$\Delta Ti = \frac{T_{hi} - T_{he}}{\varepsilon}$$

$And \quad \Delta Te = T_{he} - T_{ce}$

$Substituting\ T_{ce}\ from\ eqn\ldots(4),\ we\ have,$

$$\Delta Te = T_{he} - \left(T_{hi} - \frac{T_{hi}}{\varepsilon} + \frac{T_{he}}{\varepsilon} + C.T_{hi} - C.T_{he}\right)$$

$$\Delta Te = T_{he} - T_{hi} + \frac{T_{hi}}{\varepsilon} - \frac{T_{he}}{\varepsilon} - C.T_{hi} + C.T_{he}$$



NTU-Effectiveness Method for Parallel Flow HE

$$\Delta Te = T_{he} - T_{hi} + \frac{T_{hi}}{\varepsilon} - \frac{T_{he}}{\varepsilon} - C.T_{hi} + C.T_{he}$$

$$\Delta Te = \frac{1}{\varepsilon}(T_{hi} - T_{he}) - C(T_{hi} - T_{he}) - (T_{hi} - T_{he})$$
$$= (T_{hi} - T_{he})\left(\frac{1}{\varepsilon} - C - 1\right)$$

Heat balance eqn for hot fluid,

$$(mC_p\Delta T)_h = U.A.\Delta T_m = Q$$

$$\mathbf{(mC_p)_h(T_{hi} - T_{he}) = U.A.\frac{\Delta Ti - \Delta Te}{\ln\left(\frac{\Delta Ti}{\Delta Te}\right)}}$$

NTU-Effectiveness Method for Parallel Flow HE

$$(mC_p)_h (T_{hi} - T_{he}) = U.A. \frac{\Delta Ti - \Delta Te}{\ln\left(\frac{\Delta Ti}{\Delta Te}\right)}$$

$$(T_{hi} - T_{he}) \frac{\ln\left(\frac{\Delta Ti}{\Delta Te}\right)}{\Delta Ti - \Delta Te} = \frac{U.A}{(mC_p)_h} = NTU$$

Substituting values of ΔTi & ΔTe *in above equation, we have;*

NTU-Effectiveness Method for Parallel Flow HE

$$NTU = \frac{(T_{hi} - T_{he}) \ln \left[\frac{\frac{(T_{hi} - T_{he})}{\varepsilon}}{(T_{hi} - T_{he}) \left(\frac{1}{\varepsilon} - C - 1 \right)} \right]}{\frac{T_{hi} - T_{he}}{\varepsilon} - (T_{hi} - T_{he}) \left(\frac{1}{\varepsilon} - C - 1 \right)}$$

$$NTU = \frac{(T_{hi} - T_{he}) \ln \left[\frac{1/\varepsilon}{1/\varepsilon - C - 1} \right]}{(T_{hi} - T_{he}) \left[\frac{1}{\varepsilon} - \left(1/\varepsilon - C - 1 \right) \right]}$$

NTU-Effectiveness Method for Parallel Flow HE

$$NTU = \frac{\ln\left(\frac{1}{1 - C\varepsilon - \varepsilon}\right)}{\frac{1}{\varepsilon} - \frac{1}{\varepsilon} + C + 1} \quad OR$$

$$NTU(1 + C) = \ln\left(\frac{1}{1 - C\varepsilon - \varepsilon}\right) \Rightarrow \frac{1}{1 - C\varepsilon - \varepsilon} = e^{NTU(1+C)}$$

$$\therefore \quad 1 - C\varepsilon - \varepsilon = e^{-(1+C)NTU} \Rightarrow \varepsilon(1 + C) = 1 - e^{-(1+C)NTU}$$

Hence
$$\varepsilon = \frac{1 - e^{-(1+C)NTU}}{1 + C}$$

$$\varepsilon = \frac{T_{hi} - T_{he}}{T_{hi} - T_{ci}}$$

NTU-Effectiveness Method For Counter Flow HE

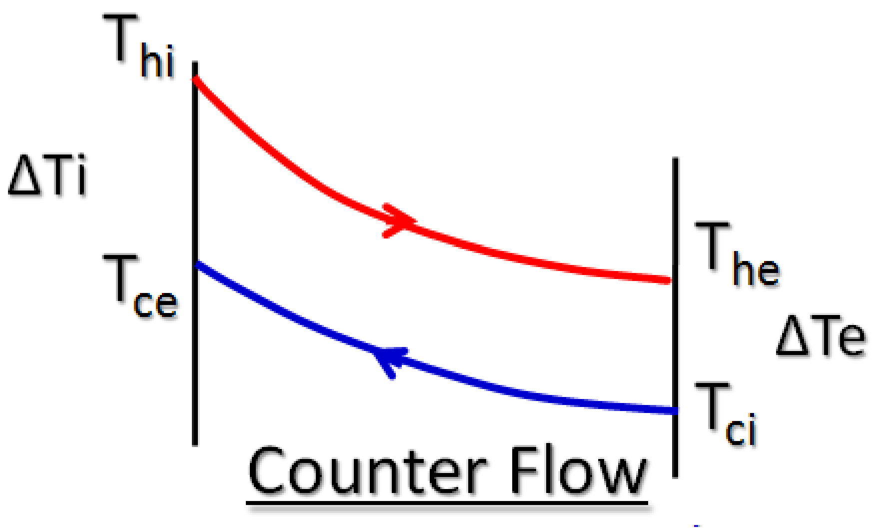
- Let $(mC_p)_h < (mC_p)_c$

$$Hence\ C = \frac{(mC_p)_h}{(mC_p)_c}$$

- Since $(mC_p)_h(T_{hi}-T_{he})=(mC_p)_c(T_{ce}-T_{ci})$

$$\therefore\ \frac{(mC_p)_h}{(mC_p)_c} = \frac{T_{ce}-T_{ci}}{T_{hi}-T_{he}} = C.......(1)$$

$$And\quad \varepsilon = \frac{(mC_p)_h(T_{hi}-T_{he})}{(mC_p)_h(T_{hi}-T_{ci})} = \frac{T_{hi}-T_{he}}{T_{hi}-T_{ci}}........(2)$$



NTU-Effectiveness Method for Counter Flow HE

Let us obtain T_{ci} & T_{ce} in terms of T_{hi} & T_{he} :

$$T_{hi} - T_{ci} = \frac{T_{hi}}{\varepsilon} - \frac{T_{he}}{\varepsilon}$$

From eqn …(2), we have,

$$\Rightarrow T_{ci} = T_{hi} - \frac{T_{hi}}{\varepsilon} + \frac{T_{he}}{\varepsilon} \ldots\ldots\ldots (3)$$

And from eqn…(1), we have, $T_{ce} - T_{ci} = C.T_{hi} - C.T_{he}$

$$\therefore \quad T_{ce} = T_{ci} + C.T_{hi} - C.T_{he}$$

$Putting\ value\ of\ T_{ci}\ from\ eqn \ldots (3)$

$$T_{ce} = T_{hi} - \frac{T_{hi}}{\varepsilon} + \frac{T_{he}}{\varepsilon} + C.T_{hi} - C.T_{he} \ldots\ldots (4)$$

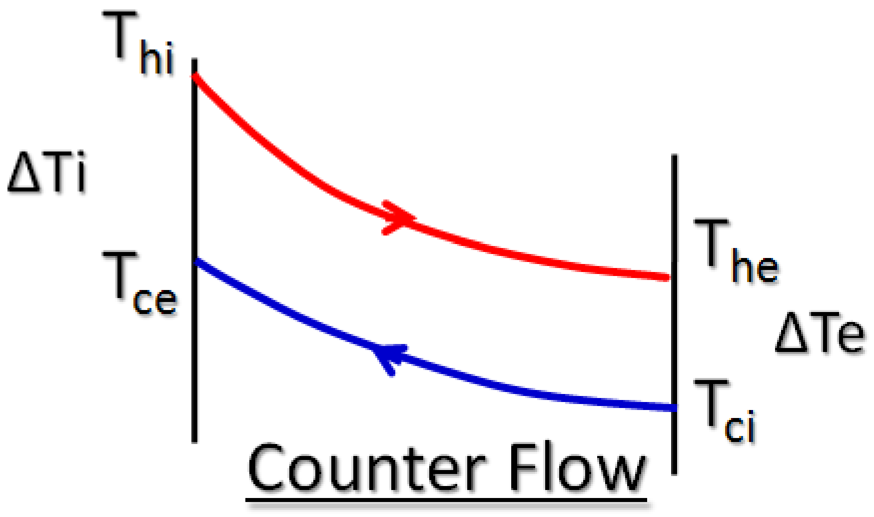
NTU-Effectiveness Method for Counter Flow HE

$Here \rightleftarrows \quad \Delta T_i = T_{hi} - T_{ce};\ Onsubstituting from\ eqn(4), wehave$

$$\Delta Ti = T_{hi} - T_{hi} + \frac{T_{hi}}{\varepsilon} - \frac{T_{he}}{\varepsilon} - CT_{hi} + CT_{he}$$

$$= \frac{(T_{hi} - T_{he})}{\varepsilon} - C(T_{hi} - T_{he})$$

$$= (T_{hi} - T_{he})\left(\frac{1}{\varepsilon} - C\right) \ldots\ldots\ldots\ldots.(5)$$



$And \quad \Delta Te = T_{he} - T_{ci}$

$Substituting\ T_{ci}\ from\ eqn\ldots(3),\ we\ have,$

$$\Delta Te = T_{he} - T_{hi} + \frac{T_{hi}}{\varepsilon} - \frac{T_{he}}{\varepsilon} = \frac{T_{hi} - T_{he}}{\varepsilon} - (T_{hi} - T_{he})$$

$$\Delta Te = (T_{hi} - T_{he})\left(\frac{1}{\varepsilon} - 1\right) \ldots\ldots.(6)$$

NTU-Effectiveness Method for Counter Flow HE

Heat balance eqn for hot fluid,

$$(mC_p\Delta T)_h = U.A.\Delta T_m = Q$$

$$(mC_p)_h(T_{hi} - T_{he}) = U.A.\frac{\Delta Ti - \Delta Te}{\ln\left(\frac{\Delta Ti}{\Delta Te}\right)}$$

$$(T_{hi} - T_{he})\frac{\ln\left(\frac{\Delta Ti}{\Delta Te}\right)}{\Delta Ti - \Delta Te} = \frac{U.A}{(mC_p)_h} = NTU$$

NTU-Effectiveness Method for Parallel Flow HE

Substituting values of ΔTi & ΔTe in above equation, we have;

$$NTU = \frac{(T_{hi} - T_{he}) \ln \left[\frac{(T_{hi} - T_{he}) \left(\frac{1}{\varepsilon} - C \right)}{(T_{hi} - T_{he}) \left(\frac{1}{\varepsilon} - 1 \right)} \right]}{(T_{hi} - T_{he}) \left(\frac{1}{\varepsilon} - C \right) - (T_{hi} - T_{he}) \left(\frac{1}{\varepsilon} - 1 \right)}$$

NTU-Effectiveness Method for Parallel Flow HE

$$NTU = \frac{\ln\left[\frac{1/_{\varepsilon} - C}{1/_{\varepsilon} - 1}\right]}{\left(\frac{1}{\varepsilon} - C - \frac{1}{\varepsilon} + 1\right)} = \frac{\ln\left[\frac{1-\varepsilon C}{1-\varepsilon}\right]}{1-C}$$

$$OR \quad \ln\left[\frac{1-\varepsilon C}{1-\varepsilon}\right] = (1-C)NTU$$

$$OR \quad \frac{1-\varepsilon C}{1-\varepsilon} = e^{(1-C)NTU}$$

NTU-Effectiveness Method for Counter Flow HE

$$OR \quad \frac{1-\varepsilon C}{1-\varepsilon} = e^{(1-C)NTU} \qquad \Rightarrow \frac{1-\varepsilon}{1-\varepsilon C} = e^{-(1-C)NTU}$$

$$OR \quad 1-\varepsilon = (1-\varepsilon C)\,.\,e^{-(1-C)NTU}$$

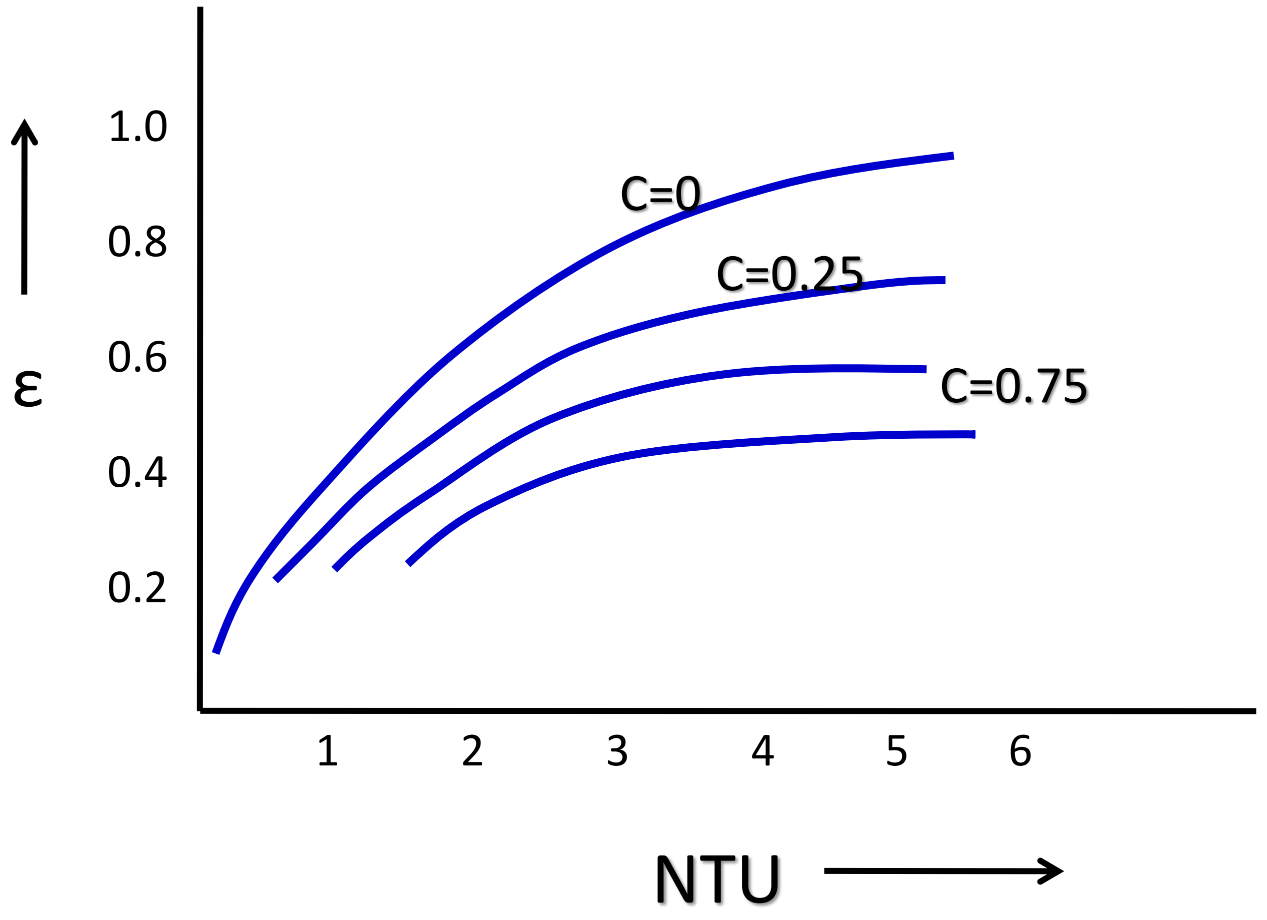
$$= e^{-(1-C)NTU} - \varepsilon C e^{-(1-C)NTU}$$

$$OR \quad 1 - e^{-(1-C)NTU} = \varepsilon - \varepsilon . C . e^{-(1-C)NTU}$$

$$= \varepsilon\left[1 - C . e^{-(1-C)NTU}\right]$$

$$\therefore \quad \varepsilon = \frac{1 - e^{-(1-C)NTU}}{1 - Ce^{-(1-C)NTU}}$$

Cross Flow Heat Exchangers



Special Cases

Case-I: C=1 & Counter Flow Arrangement ;
i.e heat capacity is same for both fluids

$$Putting\ C = 1\ in\ equation$$

$$\varepsilon = \frac{1 - e^{-(1-C)NTU}}{1 - C.e^{-(1-C)NTU}},\ we\ get\quad \frac{0}{0} \Rightarrow In \rightleftarrows \det \rightleftarrows er \rightleftarrows \min \rightleftarrows ate$$

$$Hence\ by\ applying\ L'hospital' \rightleftarrows s\ Rule,$$

$$We\ get\quad \varepsilon = \frac{NTU}{1 + NTU}$$

Special Cases

Case-II: C=0 (Condensers & Evaporators)

- When any of the two fluids changes its phase, its temp remains same; hence its heat capacity can be assumed as ∞.
- So, in case of condensers........$(mC_p)_h = \infty$; i,e $(T_{hi} - T_{he}) = 0$
- And, In case of Evaporator.......$(mC_p)_c = \infty$; i.e $(T_{ce} - T_{ci}) = 0$

$$\varepsilon = 1 - e^{-NTU}$$

- Therefore, in both the cases,
  - In such cases, flow direction of fluids is immaterial
  - For Condenser.....$(m\lambda)_h = (mC_p)_c$
  - For Evaporator.....$(m\lambda)_c = (mCp)_h$